Multiaxial strength and fatigue of rubber compounds

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Multiaxial Strength and Fatigue of Rubber Compounds

by

Joseph F Hallett

Thesis

Submitted in partial fulfilment of the requirements for the award of

Doctor of Philosophy of Loughborough University.

28th November 1997

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Abstract

Despite real applications having complex triaxial loading, current physical test methods to predict component behaviour are mainly uniaxial. But previous work has indicated that there may be substantial differences between the rubber's uniaxial and biaxial behaviour and hence through incompressibility, its triaxial properties.

In order to quantify these differences equipment was developed to assess the biaxial performance of selected rubber compounds using inflated circular diaphragms. Although allowing higher extensions than stretching a sheet in its own plane, such tests do not allow stress and strain to be measured directly, requiring careful marking of the sample, or calculation through simulation. On the grounds of perceived accuracy, the latter was chosen, requiring accurate, general, elastic constants to high extensions. In this thesis the development of this apparatus, along with the associated techniques is described, along with the development of a new elastic theory.

The tests on this new apparatus indicated significant differences between the uniaxial and biaxial strength and fatigue of rubber. In a uniaxial test natural rubber (NR) is much stronger than styrene butadiene rubber (SBR) below 35pphr of carbon black. In a biaxial test though the converse is true, although there is some evidence of crystallinity in NR during the biaxial test.

Distinct differences were also found in fatigue between the two load cases. When plotted against extension ratio the biaxial life of SBR was found to increase, while the converse is true for NR. However if life is plotted against a function of strain energy, the biaxial life of both polymers increases for a given energy.
Acknowledgements.

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To Sam and John.
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Chapter 1
Introduction and Objectives.

1.1 Reasons for Project.

The current market place for rubber components is becoming more demanding, with products evolving towards more complex geometries. This change is being driven mainly by the automotive industry, with both customers and end users placing more exacting demands on components. The main requirement is for better performance over a longer life, although rationalisation often requires many parts be replaced by one. Such changes mean more detailed information is required for more complex load conditions, increasingly required at reduced cost and development time.

The recent advent of Finite Element Analysis (FEA) codes capable of accurate, quick hyperelastic analysis has provided a useful tool in developing such products. FEA provides detailed triaxial data on the stresses and extensions in the product, a useful guide to final product performance. The accuracy of such analyses though depends on the quality of the supplied elastic deformation information.

But in order for such analyses to be meaningful, multiaxial data, both on elasticity and fatigue life are required in order to quantify the effect of the results obtained. This data also is required at extensions greater than needed for the proposed analysis, again to ensure accuracy of the predicted results. Yet current physical test methods used to assess rubber compounds and predict component behaviour are mainly uniaxial. These have several advantages as they are simple to run and to analyse, whilst elastic theories accurately match uniaxial data with relatively few constants. Multiaxial data on the other hand is more difficult to collect, especially at high extensions, and elastic constants are equally difficult to fit.

However, owing to the relative incompressibility of rubber compounds compared to their ability to distort, any two of the principal extensions are sufficient to define the distortion at any point in a rubber component. The third principal extension is given by the constancy of volume. Consequently, any improved and general laboratory physical test methods need only be biaxial in operation.
To support the need for biaxial tests, previous work \(^1,^2\) has indicated there are differences in strength and fatigue behaviour between uniaxial and biaxial testing regimes and that these differences are polymer, and possibly, filler dependent. For example, strength tests have suggested that the advantage of strain crystallising polymers over non-crystallising polymers when tested uniaxially can be lost when they are tested equibiaxially\(^1\).

The main aim of the project is to compare the uniaxial and biaxial strength and fatigue properties of practical rubber compounds.

1.2 Project Aims.

a) Test Development.
In light of the differences outlined above, and in order to enable more accurate prediction of component life, one aim of this project was to develop experimental equipment and techniques for measuring both strength and fatigue in both the uniaxial and biaxial load case.

b) Material Properties Investigation.
So that the effect of polymers and reinforcing fillers might be better understood, compounds using different polymers, carbon black types and carbon black loadings were tested. Selected to give a range of properties, these included practical compounds from industry, as well as laboratory mixed compounds based on Styrene Butadiene Rubber (SBR) and Natural Rubber (NR). These compounds formed the basis of the test development.

c) Application of FEA.
An additional aim of this project was to check the validity of the techniques developed with relation to commercial FEA packages. To do this the experimental results were compared with the FEA predictions for different elastic models.
1.3 Technique Selection.

a) Test Methodology.

The key to producing multiaxial data was the initial choice of biaxial test method. Stretching a flat sheet of rubber in its own plane to the point of failure was discounted as the clamps limit the maximum extension possible. Instead, methods based upon inflating an initially flat diaphragm of rubber were chosen. For biaxial strength, the diaphragm would be inflated to the point of rupture; whilst, for fatigue, it would be repeatedly inflated to a lesser degree until failure occurred.

In such tests, the stresses and extensions at the pole of an inflated diaphragm will be equibiaxial, whilst, adjacent to the periphery, the condition will be one of pure shear ($\lambda_2=1$ and $\lambda_3 = 1/\lambda_1$). Between these two extremes, general biaxial conditions will prevail.

It was anticipated that failure in either test, strength or fatigue, would occur at the pole of the diaphragm and therefore it would be necessary to determine the stresses and extensions at this point for a range of rubber compounds. For this, two options were available: first, measure the extensions in the region of the pole using fiducial lines marked on the diaphragm prior to inflation; or, second, employ a computer simulation of an inflating diaphragm from which the stresses and extensions could be derived. If this second option were chosen, it would be necessary to characterise the general biaxial elastic behaviour of the rubber compounds to be examined to high levels of extension.

Owing to the perceived difficulty of marking and measuring the distortion of fiducial marks, the second option was selected. Thus the characterisation of elastic properties to high extensions became an important aspect of the project.

b) Characterisation of Elasticity.

In order to undertake FEA and to aid the interpretation of experimental data, some method of accurately characterising the elasticity of the compounds was required. The required function needed to fulfil certain criteria:
• Be accurate over a large range of extensions.
• Work with the selected FEA package.
• Be accurate for all load cases.

In practice this was more complex than originally thought, but turned out to be crucial to the successful completion of the project.

1.4 Structure of Thesis.

The thesis has been divided into four main sections.

• Theoretical overview.
• Equipment design, commissioning and initial tests.
• Experimental revisions.
• Final results and discussion.
References

1 P S Oubridge, *Private communication.*

Chapter 2  
Review of Rubber Elasticity.

2.1 Statistical Treatment of Rubber Elasticity.

a) Background.

The statistical treatment of rubber elasticity\(^1\) aims to derive the elastic properties of rubber in terms of its molecular structure by considering the entropy changes undergone during deformation. The initial breakthrough came from Kuhn\(^2\) who derived a relationship between modulus and molecular weight. Further work by other authors including Treloar\(^1\) and James and Guth\(^3\) then found a relationship between stress and extension ratio.

The whole basis of this work comes from a comparison of the physical properties of a rubber with those of a gas, in particular the relationships between pressure and temperature (gas) and stress and temperature (rubber). This comparison allows certain gas like assumptions to be made:

1. The rubber is made up of \(N\) chains, where a chain is defined as the rubber molecule between two crosslinks.
2. The rubber is incompressible. i.e. \(\lambda_1\lambda_2\lambda_3 = 1\); where \(\lambda_1, \lambda_2\) and \(\lambda_3\) are the principal extension ratios (deformed lengths divided by original lengths).
3. Each chain follows Gaussian statistics, in that it is freely jointed, volumeless and the entropy of the whole network is the sum of the entropies of the individual chains.
4. The network undergoes affine deformation.

In assumption (1), the crosslinks inhibit Brownian flow, but (4) is the key. This defines the crosslinks as fixed in their mean position such that they move in the same ratio as the bulk. This was proven by James and Guth\(^3\).
b) Derivation of Statistical function.

Consider a single chain of \( n \) links of length \( l \), shown in Figure 2.1, represented by vector \( \mathbf{r}_0 \) which is deformed to \( \mathbf{r} \) (Figure 2.2), where \( x = \lambda x_0 \) etc., and \( \lambda \) is the extension.

According to Boltzmann, the entropy in a single chain, \( s_0 \), is given by:

\[
 s_0 = c - kb^2r_0^2 
\]

where \( k \) is the Boltzmann constant, \( c \) is a constant, and \( b^2 \) (the Gaussian constant) is defined as:
\[ b^2 = \frac{3}{2n.1^2} \] -2.2

Now given \( \Delta s = s - s_0 \), substituting for \( r \), and summing for all chains gives:

\[ \Delta S = -kb^2 \left\{ (\lambda_1^2 - 1)\sum x_0^2 + (\lambda_2^2 - 1)\sum y_0^2 + (\lambda_3^2 - 1)\sum z_0^2 \right\} \] -2.3

Now as the chains in the unstrained state are completely random:

\[ \sum x_0^2 = \sum y_0^2 = \sum z_0^2 = \frac{1}{3} \sum r_0^2 \] -2.4

and given that:

\[ \sum r_0^2 = N\bar{r}_0^2 \] -2.5

where:

\[ \bar{r}_0^2 = \frac{3}{2b^2} \] -2.6

which is the mean square length of the unstrained chains, assuming \( \bar{r}_0^2 \) is the same as for a corresponding set of free chains.

Combining equations 2.3 through 2.6 gives:

\[ \Delta S = -\frac{1}{2} Nk \left( \lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3 \right) \]

or:

\[ \Delta S = -\frac{1}{2} Nk(I_1 - 3) \] -2.7

where \( I_1 \) is the first strain invariant (see equation 2.16). Assuming that deformation causes no change in internal energy, we can apply the Helmholtz energy or work of deformation \( (W = -T\Delta S, \text{ where } T \text{ in temperature in K}) \) to equation 2.7, giving:

\[ W = \frac{1}{2} NkT(I_1 - 3) \] -2.8

from which a relationship for stress can be calculated.
Assuming zero deflection in the y direction ($A_2 = 0$) and zero load in the z direction ($f_3 = 0$) then:

$$dW = f_1 \, d\lambda$$  \hspace{1cm} (2.9)

and also:

$$dW = \frac{\partial W}{\partial \lambda_1} \, d\lambda_1$$  \hspace{1cm} (2.10)

where $f$ is the engineering stress (force over original cross sectional area). Equating the right hand sides of equations 2.9 and 2.10, and differentiating gives:

$$f_1 = NkT \left( \lambda_1 - \frac{1}{\lambda_1^2 \lambda_2^2} \right)$$  \hspace{1cm} (2.11)

or for true stress $\sigma$, (force divided by deformed cross sectional area):

$$\sigma_1 = \lambda_1 f_1 = NkT \left( \lambda_1^2 - \frac{1}{\lambda_1^2 \lambda_2^2} \right)$$  \hspace{1cm} (2.12)

In order to convert equation 2.12 for general use without the constraints outlined above, a hydrostatic pressure is added to all the stresses. As this pressure is not determinable, only differences between stresses are calculable, but as boundary conditions usually yield one or more values this is not a problem. For example, in the uniaxial case where:

$$\sigma_2 = \sigma_3 = 0$$

and:

$$\lambda_2^2 = \lambda_3^2 = \frac{1}{\lambda_1}$$

the stress is given by:
Chapter 2

\[
\sigma_1 = NkT \left( \lambda_1^2 - \frac{1}{\lambda_1} \right)
\]  

-2.13

c) Significance of Results and Limitations.

Equations 2.8 and 2.13, although simple, are significant as only a single elastic constant is required to represent the elastic properties of rubber in the Gaussian region. The equation accounts for increased properties with crosslink density as \( N \) increases with crosslink density, but internal energy changes are ignored. However as internal energy changes only affect the position of the tensile curve, and not its shape, such energy changes are usually compensated for when the elastic constant is fitted. It is also interesting to note that equation 2.8 is identical to the Neo-Hookean strain energy function described in equation 2.20.

The result though is usually limited to extension ratios less than 1.5, as beyond this point strain crystallisation, in crystallising polymers, and the extension limits of the individual chains, invalidate the Gaussian assumptions. Bhate et al\(^4\) have however successfully applied Gaussian network theory to two urethane based elastomers up to extensions of three uniaxially and two biaxially. These polymers do however appear to have extended linear regions.

d) Non-Gaussian Chain Statistics.

Due to the limitations of the statistical theory, further work has developed an extension to the theory to cover the non-Gaussian region. This allows for the finite extensibility of each chain, and allows for very short chain lengths (high crosslink density) which can cause problems with the Gaussian theory. Comparison of this modified form with experimental data\(^1\) shows an improved fit at higher extensions, but a poorer fit in the Gaussian region.

The improved relationship is, however, more complex and cannot be applied as widely as the statistical form. Due to this complexity, and to a lack of support in our chosen FEA package, this theory has not been examined in any detail in the work reported here, nor was fitting to experimental data attempted.
2.2 Phenomenological Treatment of Rubber Elasticity.

a) Introduction.

Owing to the limitations of the statistical theory, another approach has been employed to find a relationship between stresses and strains using continuum mechanics and the perceived properties of rubber, as opposed to its molecular structure. In order to find such a relationship, a function is required to describe the strain energy, U, stored in the system by the distortions, given by:

\[ U(\lambda_i) = \int I_i^\nu \sigma_i d\lambda_i + \int I_i^\nu \sigma_2 d\lambda_2 + \int I_i^\nu \sigma_3 d\lambda_3 \]

where \( \lambda_i \) and \( \sigma_i \) are the principle extension ratios and stresses respectively.

As this energy is a unique function of strain, it can be used to define completely the elastic characteristics of a material. First, however, certain constraints must be introduced for isotropic solids, giving rise to an expression for \( W \) in terms of Strain Invariants, \( I^{5,6} \).

\[ W = W(I_i)_{i=1,2,3} \]

where

\[ I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \]
\[ I_2 = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} + \frac{1}{\lambda_3^2} = \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_3^2 \lambda_1^2 \]
\[ I_3 = \lambda_1^2 \lambda_2^2 \lambda_3^2 = \left(\frac{\nu}{\mu}\right)^2 \]

where \( I_3 \), for an incompressible material, is equal to one.

The most general form of this function is:

\[ W = \sum_0^\infty C_{ik} (I_1 - 3)^i (I_2 - 3)^j (I_3 - 3)^k \]
Chapter 2

where $W$ is the strain energy density and $C_{ijk}$ are constants, commonly known as Rivlin constants, although for an incompressible material the function can be simplified to:

$$W = \sum_{i=0}^{\infty} C_i (I_1 - 3)^i (I_2 - 3)^j$$

-2.17

Now, if any Rivlin strain energy function ($W$) is partially differentiated with respect to the two strain invariants ($I_1$ and $I_2$), expressions are obtained for the differences between any two of the three principal true stresses:

$$\sigma_i - \sigma_j = 2\left(\lambda_i^2 - \lambda_j^2\right) \left[\frac{\partial W}{\partial I_1} + \lambda_i \frac{\partial W}{\partial I_2}\right]$$

-2.18

where $W$ is found from equation 2.17. From this equation, a relationship for simple deformations can also be calculated. For incompressible materials, subjected to either uniaxial tensile or compressive (equi-biaxial) loading, Rivlin gives the relation:

$$\frac{f}{\lambda - \frac{1}{\lambda^2}} = 2\left(\frac{\partial W}{\partial I_1} + \frac{1}{\lambda} \frac{\partial W}{\partial I_2}\right)$$

-2.19

where the left hand side is known as the 'Reduced Stress'.

However before either equation 2.18 or 2.19 can be easily used, a solution method needs to be found. This is done either by making an approximation by selecting a limited number of terms, or by substituting an empirical relationship for the infinite series for $\sigma_i$ and $\sigma_j$ by looking at the dependence of these terms on $I_1$ and $I_2$. Some authors, however, have redefined the problem in other terms rather than strain invariants.

Data is usually fitted to these equations to the chosen order of accuracy by multiple regression analysis. However it has been found that constants found to fit one type of test for a material do not always produce accurate results for another type of test. To overcome this difficulty, data from different types of tests are often combined to give a more general result. Treloar however, described this process as the "three
dimensional analogue of simple curve fitting' but, until a generalised molecular
description is found, this is a convenient mathematical representation.

b) Approximations of Rivlin Function.

i) Neo-Hookean

If the power series in equation 2.17 is truncated to only its first term, the result is the
Neo-Hookean model:

\[ W = C_{10} (I_1 - 3) \]  

which when combined with equation 2.19 gives:

\[ \frac{f}{\lambda \lambda} - \frac{1}{\lambda^2} = 2C_{10} \]  

The importance of this result lies in its similarity to the statistical function for rubber
elasticity (equation 2.8), despite being derived from a quite different starting point.
However because of this similarity, the Neo-Hookean model suffers from the same
limitations, and experimental data quickly deviates from this model with increasing
extension ratio8.

ii) Mooney-Rivlin Model.

Based on the first two terms of equation 2.17, the Mooney-Rivlin model, equation
2.22, is widely used:

\[ W = C_{10} (I_1 - 3) + C_{01} (I_2 - 3) \]  

which uniaxially equates to:

\[ \frac{f}{\lambda \lambda} - \frac{1}{\lambda^2} = 2 \left( \frac{\partial W}{\partial I_1} + \frac{1}{\lambda} \frac{\partial W}{\partial I_2} \right) \]  

One reason of its wide use is the simplicity of calculating constants from uniaxial
data. Differentiating equation 2.22 gives:
Thus if reduced stress is plotted against $\lambda$, $C_{10}$ and $C_{01}$ can be found from the intercept and slope of the resulting straight line. This linear response is adequate at low extensions, but as can be seen in Figure 2.3, at higher extensions there is a marked deviation. This deviation is less marked with unfilled polymers as the turn up occurs later, a possible explanation for Bhates\textsuperscript{4}' success with simple Gaussian theory.

Figure 2.3: Mooney-Rivlin plot for NR with 70pphr Carbon Black.

- Tension ● Compression, - - - Mooney-Rivlin Equation Fitted to Tensile Data\textsuperscript{7}.

However, as can also be seen in Figure 2.3, a model fitted to tension data does not match the behaviour in other modes of deformation, although this can be compensated for by fitting the model to data from several load cases. However this
method can be cumbersome, and data for some load cases, such as general biaxial to high strains, can be difficult to measure.

iii) Expansions of the Rivlin Function.

In order to overcome the limitations of the Mooney Rivlin model at higher extensions, extra terms may be added to give the generalised Rivlin function. Although generally improving the quality of the fit, there can be problems with such an approach, especially as accurately fitting an increasing number of constants to a given data set becomes increasingly sensitive. There will also be problems if the function is extrapolated beyond the fitted region as the shape of the curve is unknown.

Despite this, the generalised Rivlin function is commonly used, and a selection of possible functions are given in Table 2.1:

<table>
<thead>
<tr>
<th>Rivlin Expansion</th>
<th>Rivlin Elastic Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_{10}$</td>
</tr>
<tr>
<td>1st Order Invariant</td>
<td>✓</td>
</tr>
<tr>
<td>2nd Order Invariant</td>
<td>✓</td>
</tr>
<tr>
<td>3rd Order Invariant</td>
<td>✓</td>
</tr>
<tr>
<td>3rd Order Deformation</td>
<td>✓</td>
</tr>
<tr>
<td>2nd Power Invariant</td>
<td>✓</td>
</tr>
<tr>
<td>3rd Power Invariant</td>
<td>✓</td>
</tr>
</tbody>
</table>

Table 2.1: Selection of Expansions of the Mooney-Rivlin Function.

The expansion used for this project was either the 2nd or 3rd Power Invariant function as these are available in our chosen FEA package, NISA from EMRC. Also of interest is the 3rd Order Deformation function, suggested by Tschoegel, which is used as a starting point by both Yeoh and Davies et al.
iv) Yeoh\textsuperscript{7}.

It has been noted by Gregory\textsuperscript{12} that there is a simple relationship between tensile, compressive and simple shear data. It is represented by a single curve for reduced stress vs. $(I_1-3)$, which is not expected from equation 2.23. In order for this to occur, certain criteria must be matched:

$$\frac{\partial W}{\partial I_1} \gg \frac{\partial W}{\partial I_2}$$

and

$$\frac{\partial W}{\partial I_1} \text{ may not be related to } I_2.$$  \hfill (2.25)

According to Yeoh, data in the literature suggests that equation 2.25 is correct. Also, compared to $\frac{\partial W}{\partial I_1}$, $\frac{\partial W}{\partial I_2}$ is close to zero. Thus from the 3\textsuperscript{rd} order deformation function,

$$\frac{\partial W}{\partial I_2} = C_{01} + C_{11}(I_1 - 3) \approx 0$$ \hfill (2.26)

$$\therefore C_{01} = C_{11} = 0$$

Hence, Yeoh proposes the following form:

$$W = C_{10}(I_1 - 3) + C_{20}(I_1 - 3)^2 + C_{30}(I_1 - 3)^3$$ \hfill (2.27)

which for the uniaxial load case gives:

$$\frac{f}{\lambda - \frac{1}{\lambda^2}} = 2C_{10} + 4C_{20}(I_1 - 3) + 6C_{30}(I_1 - 3)^2$$ \hfill (2.28)

which is a simple quadratic in $(I_1-3)$. However the equation may need to be extended to take into account variations at low strains, for which Yeoh suggested at exponent term thus:

$$W = \frac{A}{B}(1 - e^{-B(I_1-3)}) + C_{10}(I_1 - 3) + C_{20}(I_1 - 3)^2 + C_{30}(I_1 - 3)^3$$ \hfill (2.29)
where $A$ and $B$ are constants.

Yeoh suggests that $C_{10}$ may relate to the $\frac{1}{2}NT$ term in the statistical theory and is positive, together with $C_{30}$, with $C_{20}$ typically being negative. Yeoh comments that the latter two terms correct the inaccuracies present in the statistical theory. Further, he found that this theory works well for all load conditions, which is supported by the work of Busfield et al\textsuperscript{13}, even when only fitted to one load case. However, limited trials in this project have not had the same success although the theory has not been subjected to a detailed investigation.

v) Davies, De and Thomas\textsuperscript{11}.

Using the data of Gregory\textsuperscript{12} like Yeoh\textsuperscript{7}, Davies et al aimed to solve the problems at low extensions experienced by many other theories. Although not strictly relevant to these studies, as it only works to around 100% strain, it is interesting as it contains a linear and power term like the filament theory described in Section c) ii, although based on a log-log plot.

c) Arbitrary Functions.

i) Ogden\textsuperscript{14}.

In order to simplify the mathematics, Ogden suggested a linear combination of strain invariants defined by:

$$I = \frac{\left(\lambda_1^\alpha + \lambda_2^\alpha + \lambda_3^\alpha - 3\right)}{\alpha}$$

where $\lambda_1$ to $\lambda_3$ are independent, but subject to the incompressibility condition, and $\alpha$ is a constant. This leads to:

$$\sigma_i = \lambda_i \frac{\partial I}{\partial \lambda_i} - p$$

where $p$ is a hydrostatic pressure, which for simple tension gives:
or for equi-biaxial extension:

\[ f = \sigma = \sum_r \mu_r \left( \lambda_r^{\sigma_r-1} - \lambda_r^{-(1+\sigma_r)} \right) \]

where \( \mu \) is also a constant.

One of the sponsors to this project has used this theory at high extensions on the MARC\textsuperscript{15} FEA package, but recommended the theory too late for in-depth analysis, especially as no simple analytical method of fitting the constants could be found.

ii) Filament Theory of Rubber Elasticity\textsuperscript{16}.

Proposed by Turner\textsuperscript{17}, the Filament theory is an alternative approach to describe the general elastic behaviour of rubber compounds to high strains. Developed as a tool for this project, it involves a maximum of four parameters and allows for the shape of observed uniaxial stress-strain curves. It is described in detail below and, in principle, may be applied to biaxial conditions.

Let a unit cube of rubber, Figure 2.4, when distorted, remain orthogonal so that the lengths of its sides represent the three principal extension ratios: \( \lambda_1 \), \( \lambda_2 \) and \( \lambda_3 \).

![Figure 2.4.](image)

The cube contains a single elastic filament connecting opposite corners, as shown by the thick line in the Figure. The tension in the filament, for any state of distortion, opposes the orthogonal forces applied to the faces of the cube creating the distortion. The tension, \( T \), in the filament will be a function of any pretension, \( T_o \), and the strain...
in the member, $\varepsilon$. To allow for the observed elastic behaviour of rubber compounds to high strains, the form of this function is postulated to be:

$$T = T_o + A.\varepsilon + B.\varepsilon^n$$

where $T_o$, $A$, $B$ and $n$ are fitted parameters. Now the strain induced in the filament is:

$$\varepsilon = \frac{\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2} - \sqrt{3}}{\sqrt{3}}$$

Substituting for $\varepsilon$ in equation 2.34 gives the tension ($T$) in the elastic, diagonal filament.

Now the angles ($\alpha$) the filament makes with the three principal directions are:

$$\alpha_1 = \tan^{-1}\left[\frac{\lambda_2 + \lambda_3}{\lambda_1}\right] \quad \alpha_2 = \tan^{-1}\left[\frac{\lambda_1 + \lambda_3}{\lambda_2}\right] \quad \alpha_3 = \tan^{-1}\left[\frac{\lambda_1 + \lambda_2}{\lambda_3}\right]$$

Resolving the filament tension in the principal directions gives true stresses, $\varphi$, related to the forces acting on the faces of the original unit cube:

$$\varphi_1 = \lambda_1.T.\cos(\alpha_1) \quad \varphi_2 = \lambda_2.T.\cos(\alpha_2) \quad \varphi_3 = \lambda_3.T.\cos(\alpha_3)$$

If it is assumed rubbers are incompressible, an arbitrary hydrostatic pressure ($p$) may also be applied to the faces of the cube. However, for the biaxial condition, the third principal true stress is zero. Consequently:

$$p - \varphi_3 = 0 \quad \text{or} \quad p = \varphi_3$$

Thus the actual true, biaxial stresses ($\sigma$) are:

$$\sigma_1 = \varphi_1 - \varphi_3 \quad \text{and} \quad \sigma_2 = \varphi_2 - \varphi_3$$

Thus given $\lambda_1$ and $\lambda_2$, and with $\lambda_3 = \sqrt[3]{\lambda_1 \lambda_2}$, the true biaxial stresses may be calculated knowing $T_o, A, B$ and $n$. 

Page 19
2.3 Factors Affecting Rubber Elasticity.

a) Crosslinking. In the derivation of the statistical theory, it is assumed that all the chains are load bearing. If however crosslinks do not occur at the end of chains, the last portion of the chain is ineffectual, reducing NkT and hence a. Other crosslinks may form loops in the chains which are also unproductive, reducing the strength of the material. A positive possibility is entanglements, where one chain is wrapped round another, but where no crosslink is formed. These quasi-crosslinks help to reinforce the rubber, increasing its stiffness and strength.

b) Effect of Carbon Black. Carbon black is a reinforcing filler, and for low levels (Vf<0.3), its effect on the polymer can be represented by the Guth-Smallwood equation:

\[
\frac{E_t}{E_0} = 1 + 2.5V_f + 14.1V_f^2
\]

where \(E_t\) is the reinforced modulus, \(E_0\) the gum modulus, and \(V_f\) the volume filler fraction. However, as the carbon black has a much higher modulus than that of the surrounding rubber, when the sample is deformed it can be assumed not to change. This amplifies the strain in the rubber by the relationship:

\[
\Lambda = 1 + \epsilon(1 + 2.5V_f + 14.1V_f^2)
\]

where \(\Lambda\) is the effective chain extension and \(\epsilon\) is the applied strain. As the breaking strain of the rubber molecules does not change, this reduces the extensibility of the material.

c) Effect of State of Mix. This project for the most part aimed to use well mixed compounds, but the effect of state of mix is still of interest and will be covered briefly.

When carbon black is first added to a rubber compound it is quickly incorporated into the rubber in the form of large particles or agglomerates. The agglomerates absorb
some of the matrix rubber, binding it within the carbon black structure and thus effectively increasing the carbon black loading of the material\textsuperscript{21}. However these agglomerates, unlike primary particles, are not rigidly bound together allowing them to deform with the material. Further mixing disperses the carbon black by breaking down the agglomerates releasing smaller primary particles and the occluded rubber.

The state of mix has significant effects on the mechanical properties of the polymer, and hence its elasticity. In the first instance, the poor dispersion causes a significant reduction in tear strength and hence tensile properties. However, further mixing has been\textsuperscript{19} found to cause an initial rapid increase in tear strength and tensile properties, but the effect of continued mixing varies. Tear strength was found to level off but stress, both ultimate and at a given extension, reaches a peak and then reduces slowly, the latter due to further mixing damaging the rubber matrix. Elongation at break, however, was found to increase with mixing.
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2 W Kuhn, *kolloidzschr.*, 76, p258 (1936)


9 EMRC, PO Box 696 Troy, Michigan 48099, USA


15 MARC UK Ltd., 35 Shenley Pavilions, Chalkedell Drive, Shenly Wood, Milton Keynes, MK5 6LB


17 D M Turner, *Private communication*.


3.1 Introduction.

In rubbers, as in all solids, there are inherent flaws present in the material, some microscopic, others visible. Such flaws cause a magnification of the applied stress at their tips, and fracture usually occurs at the largest stress magnification. At this point the rubber chains rupture, creating a new free surface and causing the crack to extend (known as macroscopic fracture).

This crack growth is due to a mechano-chemical process. As the sample is deformed the chains align, the load being as equally shared between them as the local topology allows. Eventually the load on one chain causes it to break, increasing the load on the other chains. This then causes the next highest loaded chain to fail, although at this point there is no macroscopic fracture.

Fracture occurs when sufficient chains break in a localised area to create new free area. This usually happens in areas of highest damage, although it has been calculated\(^1\) that for this to happen, some \(2 \times 10^6\) chains need to break for \(1 \mu m^2\) of new free area. This may take many cycles in a fatigue situation and, under certain conditions, may be delayed.

The mechanisms for these processes and methods for quantifying their effects are discussed in this Chapter.

3.2 Crack Initiation.

In all cases cracks are initiated at stress raisers in the sample. These may by microscopic such as microvoids or carbon black particles, or macroscopic such as surface defects or damage. In each case only a relatively small extension is required to start the crack\(^2\), with the extension required for crack initiation reducing with carbon black loading relative to the extension at break. It is proposed\(^2\) that this starting extension is related to the number of crosslinks, because crosslinks limit
chain extension, and the fact that the presence of carbon black increases the effective number of crosslinks.

3.3 Failure Modes.
During cut growth or fatigue, the crack propagates in various ways, with the two most common being stick slip and knotty tear. Which mechanism occurs is dependant on a variety of factors including:

- Polymer type (crystallising or non-crystallising).
- Black loading.
- Degree of cure.
- Rate of test.
- Extension at which crack growth occurs.

with the final catastrophic failure often exhibiting a different pattern.

Observations by Goldburg et al during tearing\(^2\) have indicated a cause for these patterns. During tensile testing of razor cut rectangular test pieces it was noticed that bundles of strands form at the crack tip. Goldburg suggests these are formed of uncoiling rubber molecules; and he also notes that the bundles become thicker, then finer, as the black content is increased. It is the random failure of these bundles that causes the typical pattern of a series of ridges.

The main factor, however, governing the failure mode is the type of polymer, crystallising (e.g. natural rubber, NR) or non-crystallising (e.g. styrene butadiene rubber, SBR), with the latter having time dependant crack growth and the former stick slip\(^3\). It has also been noted\(^4\) that carbon black has a greater effect on the fracture surface’s of NR than those of SBR.

Stick slip crack growth occurs in crystallising polymers, and requires a cyclic loading for crack propagation. This is beneficial as more energy is required in order to propagate the crack. This increase in tearing energy with each cycle is to overcome the increasing strength of the remaining polymer chains as their extensions increase. This energy increase continues until the ultimate strength of the chains is reached.
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Time dependant failure, which occurs in non-crystallising polymers, happens when the crack continues to grow even when the extension is constant. This is not beneficial as constant strain energy is all that is required for crack propagation.

Knotty tear though can occur with both types of polymer as it is caused by the way the crack propagates. The ridge pattern described above is caused when the crack propagates across the sample as expected, perpendicular to the applied load. With knotty tear the cracks are diverted parallel to the applied load before moving “forward”. Hamed\(^1\) suggests that this change over is governed by:

\[
G_\| < \frac{G_{\leftrightarrow}}{4\pi(1-\nu^2)} \quad \text{-3.1}
\]

where \(G\) is the fracture energy, \(\nu\) is Poisson’s ratio, and the subscripts \(\leftrightarrow\) and \(\|\) indicate across and along the loading direction respectively. The point at which this change over occurs can be affected by black loading\(^4\), strain rate\(^2\) and other factors which affect the rate of dissipation of energy from the polymer.

Finally, knotty tear is also beneficial because the crack has further to propagate, causing the rate of tear across the sample to be slower.

3.4 Fracture Mechanics.

a) Tearing Energy.

Traditionally, tear tests were used to quantify the tear strength of a material, together with various fatigue tests. However the former tests give variable results dependant on the details of the test method employed, the different methods even ranking a set of compounds in different orders. A complex finite elasticity model was considered to quantify the data obtained but, until recently, such methods have not been computationally practicable and so an energetics approach was employed.

The key to a method based upon energetics was provided by Griffith\(^5\) who was looking at crack propagation in glass. He found that a crack grew when the strain energy, \(U\), released by a crack growing is greater than the surface free energy created, \(S\), and derived the following equation:
2S = \( \left( \frac{1}{U} \right) \left( \frac{\partial U}{\partial c} \right) \)

Where \( c \) is the crack length and \( t \) the sheet thickness, whilst the subscript \( 1 \) indicates the differentiation is carried out at constant extension. This he found was material dependant but independent of test piece geometry.

Now the tearing energy, \( T \), defined as \( 2S \), can be calculated using force, extension and crack length data from a tear test. Using this data, \( U \) is calculated by integrating the force/extension plot, and \( \partial U/\partial c \) from the local slope of a plot of \( U/c \).

b) Cut Growth.

The energetics approach and the concept of tearing energy can also be applied to crack growth, and hence fatigue. Now although \( T \) varies during cyclic loading, it has been found that crack growth is determined primarily by the maximum tearing energy and is independent of wave form.

Because of this the crack growth during a cycle may be represented by:

\[
\frac{dc}{dn} = f(T)
\]

which is assumed to be of the form:

\[
\frac{dc}{dn} = BT^\beta
\]

where \( B \) is the cut growth constant, and \( \beta \) is the strain exponent.

From integration of equation 3.3, the crack growth over a number of cycles can be calculated, providing a relationship for \( T \) is known over the appropriate range. Now uniaxially, \( T \) has been shown to be given by:

\[
T = 2kWc
\]
Substitution of equations 3.4 and 3.5 into 3.3 and integration yields the number of cycles for a crack to grow. However if it is assumed that the crack propagates from an initial small flaw to a larger crack the following equation for fatigue life is found:

\[ N = \frac{1}{(\beta - 1)Bc_0^{\beta-1}(2kW)^{-\beta}} \]  

where \( N \) is the fatigue life, and \( c_0 \) is the initial flaw size, which has been shown to be of the order 20 to 60\( \mu \)m.

Roberts and Benzies have further shown that for the equibiaxial load case the fatigue life is given by:

\[ N = \frac{1}{(2\beta - 1)B'C_0^{(2\beta-1)}(2k^2W)^{-\beta}} \]  

where:

\[ B' = \frac{B\pi^\beta}{D^\beta} \]

where \( D \) describes the perimeter of the initial crack, such that \( Dc \) is its surface area.

Equations 3.6 and 3.7 are useful as they allow \( \beta \) to be found relatively simply from fatigue data, providing the strain energy density, \( k \) and \( N \) are known. However calculation of both \( B \) and \( B' \) require knowledge about the likely initial flaw size.

3.5 Factors Affecting Fatigue and Fracture.

a) Mechano-Chemistry.

Rubber networks are not only affected by their mechanical conditions, but their environment as well. Although not a problem with strength tests due to their relative speed, oxygen and ozone cause significant degradation to the properties of a rubber
compound over time. Oxygen generally causes both scission and further crosslinking, whereas ozone causes just the former.

In fatigue tests, due to their extended time, oxidation has more time to occur. This has been born out in tests comparing fatigue life in air to that in N₂, when failure has been found to occur sooner in air. There is also evidence that scission caused by oxidation is accelerated in deformed samples.

Scission is caused by the oxygen affecting the bonding of the carbon chain, with NR being especially prone, reducing the load required to rupture the chain. What happens to these chains after rupture is also governed by oxidation. In some cases the chains reform crosslinks, although this may also be prevented, both cases being caused by the by products of the oxidation reaction. Another possibility is the split chain attaches to a weakened double bond, again due to the oxidation process. Of these, the first and last reduce the effect of the oxidation as the chains are still partially load bearing.

b) Energy Dissipation.

In most cases when a chain or crosslink fails, energy is released as heat, known as catastrophic energy dissipation. If this energy, once released, is reused in reforming crosslinks or creating new links between the broken chain and its surroundings, the load in the surrounding chains is reduced. Ideally though, the crosslinks should fail before the carbon backbone of the rubber, as this leaves the rubber molecules still intact and able to form new crosslinks.

Sulphur crosslinks behave in this manner, exhibiting reversible failure at energies slightly lower than that required to fracture the rubber molecule. This failure and reformation dissipates energy, reducing the load on the chains as well as keeping all the chains in the structure load bearing. This type of failure is called non-catastrophic energy dissipation. A filler added to the polymer also introduces energy dissipating mechanisms, many of which reduce the permanent fracture of chains.
c) **Particulate Reinforcement.**

By adding reinforcing fillers, stress magnification is found to be reduced, despite higher average chain loads due to strain amplification (Chapter 2.3b and equation 2.41). It appears that these fillers non-catastrophically dissipate energy, spreading the load more easily. This process could involve ruptured chains attaching to the black’s surface, enabling them to take load again. Also intact or failed chains may slide over the surface of the black, dissipating energy through friction.

These frictional effects are important, as the energy lost through friction may even be greater than the energy required to break a chain. Also too many bonds between the rubber and black reduces strength as the black’s ability to dissipate energy is impaired. Increased interactions do however reduce the losses due to hysteresis.

d) **Strain Crystallisation.**

Present in some elastomers, strain crystallisation is a useful property which increases the strength of a material without the use of filler. This increase is due to the energy dissipation which accompanies strain crystallisation. The crystal structure of crystallised materials also need to be disrupted locally for the crack to propagate, again requiring energy.

However for fatigue, carbon black is often required to maximise performance. Despite this, crystallinity increases resistance to fatigue at high strains and if the sample is not cycled through zero. This latter case is due to the reduced risk oxidation in the crystalline form.

### 3.6 Statistical Analysis of Fatigue Data.

a) **Introduction.**

Due to the nature of fatigue crack initiation and growth, it is inevitable that a spread of fatigue lives will be observed from repeated samples. Thus some from of statistical analysis must be employed. This may be the widely used Normal statistical distribution, characterised by a mean and a standard deviation, but there are other distributions developed specifically for, or are suitable for, failure data. However all
have two functions in common, although they differ in form. They are the cumulative distribution function, \( F(x) \), Figure 3.1 and the probability density distribution function, \( f(x) \), Figure 3.2.

![Cumulative Distribution](image1)

**Figure 3.1: Cumulative Distribution.**

The Cumulative distribution function, Figure 3.1, gives the proportion of the total number of individuals which have failed with lives less than \( x \). When plotted against life, it is typically an “S” shaped curve extending from zero, when there have been no failures, up to “1”, when all the individuals have failed. On this curve, the point at \( F(x) = 0.1 \) gives the 10% quantile of the fatigue lives, \( F(x)= 0.5 \) gives the 50% quantile and \( F(x) = 0.9 \) the 90% quantile, a quantile being the life when these percentages have failed.

![Probability Density Distribution](image2)

**Figure 3.2: Probability Density Distribution**
Chapter 3

The Probability density distribution function, Figure 3.2, gives the probability of a failure occurring with a life between \( x \) and \( x + \delta x \), where \( \delta x \) is very small. With a Normal distribution, it has a symmetrical bell-shaped form if plotted against life.

b) Distributions.

i) The Normal Distribution.

The probability density distribution function of a Normal distribution is defined as:

\[
f(x) = \frac{1}{\sqrt{2\pi}b} \exp\left[ -\frac{(x-a)^2}{2b^2} \right]
\]

where \( a \) is the mean value and \( b \) is the standard deviation.

It cannot be integrated to give the cumulative distribution function but the quantiles are:

- 10% quantile = \( a - 1.282b \)
- 50% quantile = \( a \)
- 90% quantile = \( a + 1.282b \)

ii) The Kase Distribution.

The Kase distribution was developed to describe the variability experienced when measuring the tensile strength of rubber compounds. Its cumulative distribution function is defined as:

\[
F(x) = 1 - \exp\left[ -\exp\left( \frac{x-a}{b} \right) \right]
\]

where \( a \) is the characteristic life when 63.2% of individuals have failed and \( b \) is called the scale factor which is analogous to the standard deviation of the Normal distribution; the lower its value, the narrower the spread of lives.
The Kase cumulative distribution function may be differentiated to give its probability density distribution function:

\[
f(x) = \frac{1}{b} \exp \left[ \frac{x-a}{b} \right] - \exp - \frac{x-a}{b}\]

When plotted against life, this function is skewed towards the high lives.

The quantiles are:

10% quantile = \( a - 2.254b \)

50% quantile = \( a - 0.367b \)

90% quantile = \( a + 0.834b \)

iii) The Weibull Distribution.

It was reasoned that any cumulative distribution function could be written in the form:

\[
F(x) = 1 - \exp[-\Phi(x)]
\]

\( \Phi(x) \) is some function of the lives, which must be zero at some life, be positive and increase in value with x.

Such a function is:

\[
\Phi(x) = \left( \frac{x-c}{a} \right)^b
\]

so that:

\[
F(x) = 1 - \exp \left[ - \left( \frac{x-c}{a} \right)^b \right]
\]

where \( a \) is the characteristic life when 63.2% of individuals have failed, \( b \) is called the shape factor and \( c \) is a minimum life below which no failures occur.
Like the Kase, the Weibull cumulative distribution function may be differentiated to give its probability density distribution function:

\[ f(x) = \frac{b}{a^b} (x - c)^{b-1} \exp \left( -\frac{x - c}{a^b} \right) \]

When plotted against life, this function may have different forms and, because of its adaptability, is specified in some industry standards.

The quantiles are:

- 10% quantile = \( a \cdot 0.105^{1/b} + c \)
- 50% quantile = \( a \cdot 0.693^{1/b} + c \)
- 90% quantile = \( a \cdot 2.303^{1/b} + c \)

**c) Cumulative Distribution.**

When considering a Normal distribution, it is only necessary to calculate the average fatigue life, \( \bar{x} \), and the standard deviation, SD, so that:

\[ a = \bar{x} = \frac{\sum x}{n} \]

and:

\[ b = SD = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}} \]

where \( n \) is the number of failed samples.

However, with the Kase and Weibull distributions, it is necessary to establish the cumulative distribution \( F(x) \).

The \( n \) samples are first ordered from the lowest life to the highest:

\[ x_1, x_2, x_3, \ldots, x_n \]
Then, if \( n \) were very large, the cumulative function \( F(x) \) would take the values:

\[
\frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \ldots, \frac{n}{n}
\]

-3.22

In practice, these values of \( F(x) \) will be stepped estimates of a smooth curve.
Consequently, an adjustment is made. If \( n \) is less then 20:

\[
F(x_k) = \frac{k-0.3}{n+0.4}
\]

-3.23

where \( k = 1 \) to \( n \)

Also, if \( n \) lies between 20 and 100:

\[
F(x_k) = \frac{k}{n+1}
\]

-3.24

It should also be noted that if, say, there were \( m \) unfailed samples, \( n \) still equates to the total number of samples but \( k \) runs from 1 to \( (n-m) \). Thus a test may be curtailed when a relatively few samples are still running, whilst allowing for this in the analysis.

d) Fitting Distributions.

With measured fatigue lives or strengths and their cumulative distributions, there remains the task of fitting the Kase and Weibull functions to obtain their parameters. This is achieved by linearising the functions and fitting the data by regression analysis.

i) Kase.

Let:

\[
F(x) = p = 1 - \exp\left[-\exp\left(\frac{x-a}{b}\right)\right]
\]

\[
\ln(1-p) = -\exp\left(\frac{x-a}{b}\right)
\]
\[ \ln[-\ln(1-p)] = \frac{x-a}{b} \]

\[ x = b \ln[-\ln(1-p)] + a \]

which is in the form \( Y = A + M.X \)

ii) Weibull.

Let:

\[ F(x) = p = 1 - \exp \left[-\left(\frac{x-c}{a}\right)^b\right] \]

\[ \ln(1-p) = -\left(\frac{x-c}{a}\right)^b \]

\[ \ln[-\ln(1-p)] = b \ln\left(\frac{x-c}{a}\right) \]

\[ \ln(x-c) = \frac{1}{b} \ln[-\ln(1-p)] + \ln(a) \]

which is also in the form \( Y = A + M.X \)

The characteristic lives of the Kase and Weibull distributions are thus obtained directly from the regression analyses, together with the 10%, 50% and 90% quantiles.
<table>
<thead>
<tr>
<th>Reference</th>
<th>Citation</th>
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<tbody>
<tr>
<td>7</td>
<td>S Kase, <em>J.of Pol.Sc.</em>, XI No.5, p425</td>
</tr>
</tbody>
</table>
4.1 Tensile Strength Apparatus.

The tensile tester used for this project was manufactured by Hounsfield Ltd., is equipped with a laser extensometer and is computer controlled. The extensometer was required to allow measurements over the gauge section only of the test piece. The laser rapidly scans along the axis of the test piece looking for reflections from highly reflective strips adhered at two points on the gauge length. The computer then records the measured extensions and forces to plot a printable stress extension curve, together with the stresses at five chosen extensions. The latter can then be saved to disk along with the stress and extension at break.

It was difficult to find a good clamping system due to the high extensions required, but eventually the clamps shown in Figure 4.1 were found to be best.

![Tensile Test Clamp Schematic](image)

Figure 4.1: Tensile Test Clamp Schematic.

It consists of a spring loaded roller mounted on a support plate over which the rubber sample is folded. When load is applied, the roller is forced to clamp the sample against the plate giving a degree of active clamping. However, due to the small size of the samples in comparison to the rollers, great care was necessary when mounting the samples.
In order to fit elastic constants to the data saved from repeated tests, additional software was written. This calculates an average stress extension curve and the strain energy density (SED) at break. It then fits elastic constants. This software, “UNSTR.BAS”, achieved a similar function to the later “TNRHOUN.BAS” which is described in more detail in Appendix A.

4.2 Biaxial Strength Apparatus.

In order to measure the biaxial strength properties of the materials, an inflation tester was designed and built based on that used by Dunlop. The method, based upon inflating an initially flat circular diaphragm, was chosen although it is analytically more complex. Inflating a diaphragm allows operation to much higher extensions than stretching a flat sheet biaxially using apparatus with sliding clamps. In such a test using flat sheets it was reported that the tongues of the sample attached to the clamps failed at relatively low (around 150%) extensions. For the inflated diaphragm test, a 50mm diameter circular clamp was used giving equi-biaxial extension at the pole where the stresses will be maximum, although an elliptical clamp could be used to give general biaxial conditions at the pole.

On the grounds of perceived accuracy, it was decided not to mark the surface of the sample as used by other authors. It was thought that as the marks get very thick and faint at high extensions, coupled with difficulties in accurately marking before the test and measuring displacements during testing, large errors would be introduced. Instead a combination of simple dimensional measurements and pressures during the test was adopted, with the stresses calculated from a computer simulation.

The apparatus consisted of a base plate, through which the air is supplied to the centre of the sample, and a clamping ring. This is illustrated in Figure 4.2.
The air supply was controlled by a flow valve and the air pressure in the sample was measured from the supply pipe, originally using an accurate pressure gauge connected through a non-return valve. This, however, proved to be inaccurate and a pressure transducer was substituted instead, without the non-return valve. The pressure was then measured as a voltage (1 volt indicating a pressure of 1 bar) using a digital voltmeter capable of storing the maximum value, accurate to 1mV. Physical dimensions of the inflated diaphragm were measured from a video recording of a test.

Originally, image analysis software was used to obtain the dimensions of the bubble from the video, however this proved to be difficult and inaccurate. Instead graph paper was placed behind the test, as shown in the Figure 4.2. The video can then be watched and the dimensions of interest measured from the graph paper. Also by carefully placing the voltmeter, the pressure at each height was recorded on the video. The dimensions of interest are the height, $H$, of the diaphragm above the clamp; its width, $W$; and the height at which the maximum width occurs, $H@W$. These dimensions are shown in Figure 4.3.
As stated earlier, the chosen method required computer simulation to analyse this data. This software uses an iterative finite element technique to calculate the stresses and extensions around the diaphragm as it is inflated. The data required include both the elastic constants derived from the uniaxial test, and the measured diaphragm dimensions. The software, "SIMDIA.BAS", is described in more detail in Appendix A.

4.3 Uniaxial Fatigue Apparatus.

The uniaxial fatigue machine was specified by the author but designed and built by Hampden Test Equipment Ltd. It allows eight BS type 2 dumbbells\(^4\) to be tested simultaneously at extension ratios from 1 to 4 (0 to 300\%), although the highest extension is only possible with 4 samples owing to the available power from the motor. It is driven by an electrical motor with automatic counting and failure detection. The apparatus is shown in Figure 4.4.
In order to adjust the extension ratio, a variable position cam is used in conjunction with various sized spacers fitted to the push rod, as shown in Figure 4.4. Also the motor speed can be set using a potentiometer to give the desired angular velocity, displayed by a speed indicator.

Both the top and bottom clamps used in this apparatus use a metal plate, which is screwed down by two screws to fix the sample. The top clamp has adjusters to take up any permanent set in the samples and the bottom clamps are allowed limited movement on bronze bushes as part of the failure detection mechanism. This uses a short spur mounted on the back of the clamp (see Figure 4.5) with a LED and detector. At top dead centre the lower clamps will be raised if the sample is intact, and the detector will ‘see’ the LED. If this occurs, the counters are incremented, otherwise the sample is assumed to have failed. When all samples are deemed to have failed the motor is automatically stopped. The bottom clamp is shown in Figure 4.5.
4.4 Biaxial Fatigue Apparatus.

The last major piece of apparatus to be developed is the biaxial fatigue tester built by HNL Instruments and Controls Ltd. As with the biaxial strength tester it was specified by the author and is based on a Dunlop design using inflated diaphragms. It is completely pneumatic in operation. It also tests 8 samples in two independent banks of four, with automatic counting and failure detection. The apparatus is shown in Figures 4.6 and 4.7.

Figure 4.6: Biaxial Fatigue Machine Control and Logic.
In Figure 4.6 may be seen the pressure regulators used to set the inflation pressure, the counters and oscillator controls. The oscillator controls the cycle frequency, as well as the "width" of the "on" pulse. This is important and the failure detection system checks that the sample touches the height sensors within the "on" pulse, otherwise the air is switched off to that sample after a few attempts.

![Figure 4.7: Biaxial Fatigue Machine Sample Station.](image)

The height sensors are located on an adjustable bar, as shown in the Figure 4.7, which is adjusted using a vernier slide. The sensors consist of a nozzle through which air is passed and which creates a back pressure when touched. This is amplified and passed back to the control logic, which then increments the counter.

The clamps used are similar to the biaxial strength tester, being 50mm diameter, but thicker. Finally a pressure transducer may be connected as shown to allow measurement of the pressure cycle for comparison with the strength test result.

With this type of experiment using repeated cycles of compressed air, temperature effects due the repeated compression or decompression of the gas may be a problem. Although this was unlikely to have occurred with this type of apparatus, or have any effect on the polymers used in this work, it may need to be considered in future tests. A very simple example of the possible magnitude of the temperature change is given in Appendix F.
References

1 P S Oubridge, *Private Communication*


5.1 Tensile Strength Tests.

a) Test method.

Before inserting the sample, the average thickness of the gauge length was taken from measurements at three locations, and reflective strips were placed on the sample between 10 and 19mm apart. The samples were then mounted round the clamps, trying not to pre-stretch less stiff materials. There were three main problems associated with this process:

- Placing the sample centrally or evenly round the clamp, as placing samples off centre caused the applied load vector to be slightly off the centre of the sample.

- Pre-stretch, as ordinary cut samples were only just long enough to be placed round the rollers.

Once the samples were mounted, the laser was checked to see that it was detecting the two markers on the sample and not getting spurious reflections from parts of the tester. This was done by placing a finger over each reflective strip in turn and checking that the ‘only one benchmark’ error appeared on the display panel. The test data could then be entered into the computer and the test run.

The data required by the computer is as follows:

- First a description of the sample (name etc.), operating conditions (temperature and humidity). These are optional.

- Second the test speed required, the limits for the display (the maximum stress and strain), and the five extension points of interest.

- Finally the width and thickness of the test piece.

The display limits and five extension ratios of interest are important for the correct analysis of the test. The display limits need to be large enough to accommodate the
test as the system creates an error when the limits are reached, losing all the test data. The five extension points also need to be carefully chosen as these are the basis of later analysis. It was found that these needed to be spaced fairly evenly along the stress/strain curve, and not congregated around a specific area. However, a small degree of latitude was found to be acceptable to get more detail around a specific point.

Once the test was completed, the stress strain curve and results was printed on the attached printer and the results saved to disk.

b) **Data Preparation.**

Once a run of repeated tests was completed on a compound, the output files from the test needed to be converted to ASCII files using a file utility program requested from Hounsfield. This ASCII file could then be read by the analysis software “UNSTR.BAS”. This prompts for the material type, and then the file name of the test. The five extension ratios chosen are then entered, with defaults suggested for each compound type, along with the thickness of the sample. A series of curves are then fitted through the data, enabling selection of the best fit.

c) **Chosen Elastic Model.**

For a number of reasons, it was decided to use the Filament theory to characterise the elastic behaviour of the compounds studied to high strains. These included:

- The inability of the Mooney-Rivlin strain energy function (2 elastic constants) to simulate the inflation of circular diaphragms (see Chapter 7.3).

- The lack of general biaxial data to allow the fitting of a Rivlin function with three or more elastic constants.

- The ease by which the Filament parameters could be fitted to uniaxial data, with the possibility of employing a correction factor to allow for biaxial behaviour (see Chapter 7.3 b)).
d) Data Analysis.

Once the test run was analysed by "UNSTR.BAS", the results file were examined statistically and entered into Excel, a spreadsheet supplied by Microsoft. The results files were also used to fit Filament constants using "TPROPS.BAS". It was found that the best method of using "TPROPS.BAS" was to estimate a set of constants and allow the software to iterate around these values. The fit could then be manipulated graphically, and another iteration performed using the modified values.

5.2 Biaxial Strength Tests.

a) Test method.

i) Calibration.

Owing to parallax error, before any testing could be carried out, the biaxial tester's video measurement system required calibration. To do this indicated widths were compared to a horizontal scale placed at several heights on the centre line of the sample (Figure 5.1) to give a width calibration. The procedure was then repeated for height by comparing heights at various widths. These results were then plotted (real against actual) and a correction factor calculated from the gradient. These factors were then averaged for the height and width calibration factors (typically between 12% and 15% depending on camera position).
ii) Test Method.

After taking an average thickness of the sheets from several measurements at different places, test pieces to fit the clamp cavity were cut. These were then clamped to the tester, tightening the screws progressively. Highly elastic materials, however, were found to require additional clamping to form a seal, and an O-ring placed between the sample and clamping plate was found to suffice. The voltmeter was then set to record the maximum voltage, and a zero pressure reading taken.

Pressure was then applied, with the rate controlled to give the same rate of inflation for each test, although this was difficult to achieve in practice. During each test the height was monitored by watching the video screen so a rough estimate of the height at burst could be made. Every other test though was recorded for more accurate analysis, with the test number and material noted and placed in a visible position.
iii) Measurements.

For each test, the maximum stored pressure and an estimate of the maximum height reached was recorded, but the videoed tests allowed the following to be measured at various heights to burst:

- height,
- width,
- height at which this width occurred (height at width, H@W),
- pressure,
- time taken to this height,

as defined in Figure 4.3, with all dimensions being corrected for parallax.

b) Data Analysis.

Analysis of the biaxial results proved difficult, as physical properties (i.e. stress or strain) were not being measured directly. To overcome this some software, "SIMDIA.BAS" was written (Appendix A) to calculate the extension ratio at the pole by matching the experimental and predicted heights. However the predicted pressures (and hence stresses) calculated by "SIMDIA.BAS" proved to be inaccurate (see Chapter 7.3b), requiring the stresses to be calculated from the theory of a doubly curved membrane¹:

For an arbitrary section of curved surface, Figure 5.2, its four edges s₁ and s₂ have radii of R₁ and R₂ and tensions per unit length of T₁ and T₂ respectively.
Now the component of forces along the two edges $s_2$ normal to $s_1$ (Figure 5.3) are given by equation 5.1:

$$F_1 = 2(T_1 s_2) \sin \frac{\phi_1}{2}$$

-5.1

As can be seen in Figure 5.3,

$$\sin \phi_1 = \frac{s_1}{R_1}$$

-5.2

Assuming small angles, $\sin \phi = \phi$, thus from equation 5.2, equation 5.1 becomes;
\[ F_1 = T_1 s_2 \frac{s_1}{R_1} \]

and the component of forces along the two edges \( s_1 \), normal to \( s_2 \) is;

\[ F_2 = T_2 s_1 \frac{s_2}{R_2} \]

Now the sum of the normal forces will equal that due to the pressure, hence

\[ T_1 s_2 \frac{s_1}{R_1} + T_2 s_1 \frac{s_2}{R_2} = P s_1 s_2 \]

Hence;

\[ P = \frac{T_1}{R_1} + \frac{T_2}{R_2} \]

At the pole, the radii and tensions are equal, and given

\[ \sigma = \frac{T}{t} \]

where \( t \) is the deformed thickness, equation 5.6 can be rearranged thus,

\[ \sigma = \frac{P R}{2t} \]

now

\[ \frac{t}{t_0} = \lambda_3 = \frac{1}{\lambda^2} \]

assuming incompressibility, where \( t_0 \) is the original thickness. Hence

\[ \sigma = \lambda^2 \frac{P R}{2t_0} \]
5.3 Uniaxial Fatigue Tests.

Before clamping samples in the uniaxial fatigue machine, the throw needed to be set for the required extension, compensating for the fact that not only the gauge length is extended. From early FEA investigation on the effect of different elastic constants on a dumbbell model (see Chapter 12.2), it was found that on average 73% of the overall extension takes place in the gauge length regardless of the constants used. Because of this it was decided to use 73% as a simple method to give the required extension in the gauge length. Once this was known, the throw was set to this figure, less 50mm, the distance between the clamps. This was done by adjusting the cam to the right value, and adding spacers half the throw thick as shown in Figure 4.4.

Once the throw was set, and the cam reset to bottom dead centre, the jaw separation was checked and the samples inserted. To do this the samples were clamped at the bottom first, ensuring they were central in the clamp and that, when tightened, they did not hinder operation of the failure sensor. The top clamp was then tightened, again ensuring the sample was central.

Once all the samples were secured the motor was set at 1Hz as this was similar to the maximum biaxial fatigue test speed, and the motor started. After 100 cycles the preset was removed by adjusting the top clamps until the bottom clamps were all raised to the top of the posts again. The motor was then restarted until all the samples failed.

Analysis on the failure data was then carried out using the statistical methods outlined in Chapter 3.6.

5.4 Biaxial Fatigue Tests.

For the biaxial fatigue tester, samples approximately 7cm square were cut from nominally 1mm thick sheet and an average thickness measured using the height sensors, which compensates for differing sensor sensitivity. The samples were then clamped down, tightening the screws progressively, although in this case no additional clamping ‘O’ ring was required for any material.

Once the material was clamped, the required height was set, adding the measured thickness, and the inflation pressure adjusted so the sample just touched the sensor,
triggering the counter. The apparatus is then set to auto, and the oscillator switched on, adjusting the frequency to give a slight pause between each cycle.

Once the samples had reached 50 cycles, the oscillator was stopped, and the height of the pre-set measured, again using the height sensors. The test was then restarted until all the samples have failed. However, during the test, the pressure setting sometimes needed to be revised to prevent over inflation as the test progressed.

Analysis on the failure data was then carried out using the statistical methods outlined in Chapter 3.6.
References

Chapter 6
Sample Preparation.

6.1 Compounding Details and Nomenclature.

For this project two main series of compounds were tested to compare crystallising and non-crystallising polymers under multi-axial loading. For this NR and SBR compounds were mixed with various amounts of N330 and N660 grades of carbon black, as well as some commercial compounds from the sponsoring companies in the initial tests. One of these compounds, SBR with 35pphr N330, was also used to assess the effect of state of mix and cure on material properties (referred to in this thesis as state of mix tests). The other ingredients for these experimental compounds are listed in Table 6.1;

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>NR (SMR 10)</th>
<th>SBR (Intol 1500)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NR (SMR 10)</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>SBR (Intol 1500)</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Zinc Oxide</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Stearic. Acid</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Santocure\†</td>
<td>0.7</td>
<td>1.2</td>
</tr>
<tr>
<td>Perkocit DPG grs\‡</td>
<td>-</td>
<td>0.2</td>
</tr>
<tr>
<td>Sulphur</td>
<td>3.1</td>
<td>1.8</td>
</tr>
<tr>
<td>Santoflex IPPD\§</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 6.1: Compounding Ingredients

Throughout this thesis the compound details will be abbreviated. For the commercial compounds this will be based on the company name whereas, for the experimental compounds, it is based on the polymer, carbon black loading and type. A complete list of these compound codes, including those supplied from the sponsoring companies are listed in Table 6.2;

\† Cyclohexylbenzothiazole-2-sulphenamide (CBS), Flexys.

\‡ Diphenylguenidine (DPG), Flexys.

\§ N-Isopropyl-N-phenyl-p-phenylenediamine (IPPD), Flexys.
### Table 6.2: Compound Codes for Tested Materials.

<table>
<thead>
<tr>
<th>Code</th>
<th>Compound</th>
<th>Supplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>N0</td>
<td>NR Gum</td>
<td>Mixed</td>
</tr>
<tr>
<td>N23</td>
<td>NR 20pphr N330</td>
<td>Mixed</td>
</tr>
<tr>
<td>N33</td>
<td>NR 35pphr N330</td>
<td>Mixed</td>
</tr>
<tr>
<td>N53</td>
<td>NR 50pphr N330</td>
<td>Mixed</td>
</tr>
<tr>
<td>N26</td>
<td>NR 20pphr N6600</td>
<td>Mixed</td>
</tr>
<tr>
<td>N46</td>
<td>NR 40pphr N6600</td>
<td>Mixed</td>
</tr>
<tr>
<td>N66</td>
<td>NR 60pphr N6600</td>
<td>Mixed</td>
</tr>
<tr>
<td>S13</td>
<td>SBR 10pphr N330</td>
<td>Mixed</td>
</tr>
<tr>
<td>S23</td>
<td>SBR 20pphr N330</td>
<td>Mixed</td>
</tr>
<tr>
<td>S33</td>
<td>SBR 35pphr N330</td>
<td>Mixed</td>
</tr>
<tr>
<td>S53</td>
<td>SBR 50pphr N330</td>
<td>Mixed</td>
</tr>
<tr>
<td>S16</td>
<td>SBR 10pphr N6600</td>
<td>Mixed</td>
</tr>
<tr>
<td>S26</td>
<td>SBR 20pphr N6600</td>
<td>Mixed</td>
</tr>
<tr>
<td>S46</td>
<td>SBR 40pphr N6600</td>
<td>Mixed</td>
</tr>
<tr>
<td>S66</td>
<td>SBR 60pphr N6600</td>
<td>Mixed</td>
</tr>
<tr>
<td>AE</td>
<td>EPDM</td>
<td>Avon</td>
</tr>
<tr>
<td>ANB</td>
<td>50/50 NR/SBR Blend</td>
<td>Avon</td>
</tr>
<tr>
<td>DNe</td>
<td>Neoprene</td>
<td>Dunlop Oil and Marine</td>
</tr>
<tr>
<td>DNi</td>
<td>Nitrile</td>
<td>Dunlop Oil and Marine</td>
</tr>
<tr>
<td>DSC</td>
<td>50/25/25 SBR/BR/NR Blend</td>
<td>Dunlop Suspensions and Components</td>
</tr>
<tr>
<td>P</td>
<td>45/55 NR/BR Blend</td>
<td>Pirelli</td>
</tr>
<tr>
<td>PU</td>
<td>Polyurethane</td>
<td>London International</td>
</tr>
<tr>
<td>LIG</td>
<td>Latex</td>
<td>London International</td>
</tr>
</tbody>
</table>

#### 6.2 Mixing Procedure.

The experimental compounds described in Table 6.1 were all mixed in a Francis Shaw K1 Intermix internal mixer with a capacity of 5.5 litres. This is computer controlled, with a facility to log mixing conditions throughout the mixing cycle, including batch temperature...
and power input. After mixing the compound was dumped onto a 12” long by 6” diameter 2 roll mill, pre-heated to 60°C.

The mixing cycles used for this project were based on previous work\(^1\) at the University. Despite this, some iteration was required to get the fill factor, initial temperature and mixing times correct to prevent scorch and give a good state of mix. The final mixing times are listed in Tables 6.3a and 6.3b, with the operating conditions in Table 6.4.

<table>
<thead>
<tr>
<th>Stage</th>
<th>NR</th>
<th>SBR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polymers</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Carbon black</td>
<td>180</td>
<td>90</td>
</tr>
<tr>
<td>Curatives</td>
<td>420</td>
<td>335</td>
</tr>
<tr>
<td>Discharge</td>
<td>510</td>
<td>425</td>
</tr>
</tbody>
</table>

Table 6.3a: Mixing Times for Experimental Compounds

<table>
<thead>
<tr>
<th>Stage</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Moderate</td>
</tr>
<tr>
<td>Polymer</td>
<td>0</td>
</tr>
<tr>
<td>Carbon black</td>
<td>90</td>
</tr>
<tr>
<td>Curatives</td>
<td>210</td>
</tr>
<tr>
<td>Discharge</td>
<td>300</td>
</tr>
</tbody>
</table>

Table 6.3b: Mixing Times for State of Mix Tests

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fill Factor</td>
<td>0.5</td>
</tr>
<tr>
<td>Motor Speed</td>
<td>40rpm</td>
</tr>
<tr>
<td>Coolant Temperature</td>
<td>40°C</td>
</tr>
<tr>
<td>Initial Mixer Body Temperature</td>
<td>70°C</td>
</tr>
</tbody>
</table>

Table 6.4: Mixer Operating Conditions.
Before a series of mixes, or when changing polymers, a warm up batch was put through the mixer. This had two purposes: to warm up the mixer to over the required starting temperature; and to remove any residue of previous compounds. After this and subsequent batches the mixer body temperature was then allowed to cool to the selected minimum mixer body temperature of 70°C before starting the next batch.

For each batch, the mixing cycle was logged to monitor any major differences in mixing profiles between batches. This was especially important for the state of mix tests as the aim was only to change the final dispersive mixing time. An example of a typical mixing trace is shown in Figure 6.1, with the key stages labelled.

![Figure 6.1: Typical Mixer Power Trace](image)

After discharge, the compound was passed through the two roll mill, banding the mix for 3 or 4 revolutions to give an even thickness to assist moulding. In general the nip was set to give sheet thickness no more than 1mm above the required moulded thickness to reduce flow problems in the mould. This was not however always possible.
6.3 Sample Curing.
Once milled, the rubber compound was left to rest overnight before assessing the cure time required. This was done using a Wallace Precision Cure Analyser. After checking for scorch, and ensuring the cure time was reasonable (5 to 20 minutes) at the chosen temperature, the samples were cured in a double daylight hydraulic press with square 35cm platen. Pressurised by a hydraulic motor, the press was capable of exerting 40 tonnes of force on a 20cm diameter ram, although only 20 tonnes force was applied. Temperature was controlled by electronic thermostats on each platen connected to a thermocouple.

To cure the rubber three moulds were used: a 2mm thick 120mm square sheet mould for dumbbells, each sheet producing 10 BS type 2 dumbbells; a large 1mm thick 220mm square mould for biaxial tests; and a 4 dumbbell mould. This last mould was made for the project, and was designed both to reduce flash on the samples, and remove possible edge defects introduced by cutting samples. The dumbbells also had larger tabs than the usual cut BS type 2 dumbbell, 25mm compared to 12.5mm, to ease mounting on the tensile tester.

6.4 Determination of State of Mix.
In order to assure an even level of dispersion for all mixed compounds, the state of mix (level of dispersion) was assessed for selected compounds, including all the S33 compounds used in the investigation of state of mix. In order to assess dispersion, 1µm thick sections of cured compound were taken by glass knife microtomy and analysed as proposed by Clarke and Frealdey.

These thin sections were then examined in several places, and the area covered by agglomerates, which show as much darker irregular shapes, were measured using a computer connected to a microscope. This was achieved by drawing round the agglomerates using a graphics tablet with the computer calculating the area of the present agglomerate, and the total area of measured agglomerates. This total area of agglomerates was then compared with the total area analysed, giving an area fraction of agglomerates. As the thickness of the thin section was so small, it was considered reasonable to assume the measured area fraction of agglomerates was equivalent to the volume fraction of
agglomerates, \( \phi_a \), (0% for a perfect mix). The effective volume fraction of filler, \( \phi_e \), is given by the following equation\(^3\):

\[
\phi_e = (\phi_a \times \alpha) + \phi_t
\]

where \( \phi_t \), the true volume fraction of the carbon black calculated from its true density (1.8 g/cm\(^3\)), equation 6.2;

\[
\phi_t = \frac{\text{pphr}}{\rho_{CB} \cdot (100/\rho_{RU})}
\]

where \( \rho_{CB} \) and \( \rho_{RU} \) are the densities of carbon black and polymer respectively.

\( \alpha \) is the volume fraction of immobilised rubber in an agglomerate. This is determined from the Dibutyl phthalate adsorption (DBPA) value of the carbon black using equation 6.3;

\[
\alpha = \frac{\text{DBPA}}{\text{DBPA} + 100/\rho_{CB}}
\]

An effective carbon black loading, pphr\(_e\), can also be calculated taking into account the immobilised rubber which acts as part of the filler and hence reduces the volume of effective rubber;

\[
\text{pphr}_e = \frac{\rho_{CB} \cdot \phi_e}{\rho_{RU} \cdot (1 - \phi_e)}
\]
References

1 J Clarke, Private Communication.


Chapter 7
Initial Tests and Equipment Commissioning.

7.1 Introduction.

The work carried out in the first instance was primarily used to guide the more detailed work later in the project. As part of this process the new apparatus was commissioned and the test methodologies developed. This required the development of the analysis tools and software. Also the initial tests were used to guide material selection for the later experimentation from the initial wide range.

In order to do this, a comparison of carbon black types was undertaken, as well as a brief investigation with some industrial compounds. The effect of test rate and cut and moulded dumbbells on tensile strength were also examined, the latter to assess the most reproducible method of producing samples for tensile testing. This chapter contains a brief overview of these results, with certain key results being described in more detail.

In this Chapter this development work is split into three main sections: Test Methodology Trials, where the results which helped select future test methods are described; Material Representation, where initial attempts at modelling the elastic behaviour of the materials is described; Results where the initial key results are given in detail, along with a brief summary.

7.2 Test Methodology Trials.

a) Comparison of Cut and Moulded Samples.

At the start of the project it was decided to reduce the chance of edge defects affecting the results, especially those from the uniaxial fatigue tests. To do this a mould was manufactured for individual dumbbells. However as the process of moulding dumbbells was more labour intensive, it was decided to determine whether there was any difference in uniaxial strength by testing three of the sponsors compounds as both cut and moulded dumbbells. It was expected the moulded samples would have better tensile properties due to the lack of flaws from the cutter and the fact that all the filler should be enclosed, both of which will reduce the possibility of edge defects.

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In Table 7.1 to Table 7.4 the results are compared. The significance of differences figure must however be greater than 95%, and preferably greater than 97.5%, to be considered significant. Analysis of variance with replication was used for the statistical analyses.

### Table 7.1: Extension Ratio at Failure - Cut v. Moulded

<table>
<thead>
<tr>
<th>Compound</th>
<th>Ref.</th>
<th>D.of F.</th>
<th>Mean</th>
<th>St.Dev.</th>
<th>D.of F.</th>
<th>Mean</th>
<th>St.Dev.</th>
<th>Sig.of Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPDM</td>
<td>6</td>
<td>6</td>
<td>4.86</td>
<td>0.31</td>
<td>7</td>
<td>4.38</td>
<td>0.20</td>
<td>99.7%</td>
</tr>
<tr>
<td>DSC</td>
<td>6</td>
<td>6</td>
<td>6.05</td>
<td>0.30</td>
<td>5</td>
<td>6.33</td>
<td>0.18</td>
<td>92.2%</td>
</tr>
<tr>
<td>PIRELLI</td>
<td>7</td>
<td>6</td>
<td>6.32</td>
<td>0.30</td>
<td>7</td>
<td>5.97</td>
<td>0.65</td>
<td>80.0%</td>
</tr>
</tbody>
</table>

### Table 7.2: Engineering Stress at Failure - Cut v. Moulded (MPa)

<table>
<thead>
<tr>
<th>Compound</th>
<th>Ref.</th>
<th>D.of F.</th>
<th>Mean</th>
<th>St.Dev.</th>
<th>D.of F.</th>
<th>Mean</th>
<th>St.Dev.</th>
<th>Sig.of Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPDM</td>
<td>6</td>
<td>6</td>
<td>12.7</td>
<td>0.66</td>
<td>7</td>
<td>12.5</td>
<td>0.59</td>
<td>54.2%</td>
</tr>
<tr>
<td>DSC</td>
<td>6</td>
<td>6</td>
<td>14.2</td>
<td>0.74</td>
<td>5</td>
<td>13.7</td>
<td>0.46</td>
<td>80.8%</td>
</tr>
<tr>
<td>PIRELLI</td>
<td>7</td>
<td>7</td>
<td>18.5</td>
<td>1.35</td>
<td>7</td>
<td>17.2</td>
<td>2.67</td>
<td>95.0%</td>
</tr>
</tbody>
</table>

### Table 7.3: True Stress at Failure - Cut v. Moulded (MPa)

<table>
<thead>
<tr>
<th>Compound</th>
<th>Ref.</th>
<th>D.of F.</th>
<th>Mean</th>
<th>St.Dev.</th>
<th>D.of F.</th>
<th>Mean</th>
<th>St.Dev.</th>
<th>Sig.of Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPDM</td>
<td>6</td>
<td>6</td>
<td>62.0</td>
<td>7.00</td>
<td>7</td>
<td>54.7</td>
<td>4.56</td>
<td>97.0%</td>
</tr>
<tr>
<td>DSC</td>
<td>6</td>
<td>6</td>
<td>86.2</td>
<td>8.02</td>
<td>5</td>
<td>86.9</td>
<td>3.54</td>
<td>14.9%</td>
</tr>
<tr>
<td>PIRELLI</td>
<td>7</td>
<td>7</td>
<td>117.0</td>
<td>13.0</td>
<td>7</td>
<td>96.8</td>
<td>29.3</td>
<td>90.6%</td>
</tr>
</tbody>
</table>

### Table 7.4: Strain Energy Density at Failure - Cut v. Moulded (MJ/m³)

<table>
<thead>
<tr>
<th>Compound</th>
<th>Ref.</th>
<th>D.of F.</th>
<th>Mean</th>
<th>St.Dev.</th>
<th>D.of F.</th>
<th>Mean</th>
<th>St.Dev.</th>
<th>Sig.of Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPDM</td>
<td>6</td>
<td>6</td>
<td>28.5</td>
<td>3.77</td>
<td>7</td>
<td>24.1</td>
<td>2.41</td>
<td>98.2%</td>
</tr>
<tr>
<td>DSC</td>
<td>6</td>
<td>6</td>
<td>30.8</td>
<td>3.06</td>
<td>5</td>
<td>30.8</td>
<td>1.24</td>
<td>3.9%</td>
</tr>
<tr>
<td>PIRELLI</td>
<td>7</td>
<td>7</td>
<td>39.3</td>
<td>4.78</td>
<td>7</td>
<td>32.3</td>
<td>9.85</td>
<td>90.7%</td>
</tr>
</tbody>
</table>
It can be seen here that only EPDM shows any significant differences between the cut and moulded samples, noting that the true stress and strain energy density both reflect the extension ratio difference. However it can also be seen that the cut results were higher than the moulded results, not as expected. This unexpected behaviour could be due to problems with cure as the dumbbell mould is much thicker or, more likely, defects in the moulded samples introduced when flash was removed. Despite the mould being designed to be flashless, flash was still present. Unfortunately it appears that removing all the flash back to sample may damage the edge, negating any benefits of moulding. Despite this, it was decided future tests should use moulded dumbbells, with more care taken when removing excess flash. This decision was dictated by the reduced scatter of fatigue test data (see Section 7.4).

b) **Effect of Rate on Strength Test Results.**

In terms of the effects rate may have on the behaviour of polymer-carbon black structures, the range of extension rates on the Hounsfield tensometer is narrow. Nevertheless, a limited number of test pieces were examined at extension rates from 100 mm/min to 400 mm/min. Table 7.5 lists the results for the Dunlop S&C blend at two rates; whilst Table 7.6 is for the Dunlop O&M's Nitrile compound at three rates.

<table>
<thead>
<tr>
<th>Measured Property</th>
<th>Extension Rate</th>
<th>Significance of Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>200mm/min</td>
<td>400mm/min</td>
</tr>
<tr>
<td></td>
<td>mean</td>
<td>SD</td>
</tr>
<tr>
<td>Extension Ratio (-)</td>
<td>6.16</td>
<td>0.05</td>
</tr>
<tr>
<td>Eng. Stress (MPa)</td>
<td>14.4</td>
<td>0.17</td>
</tr>
<tr>
<td>True Stress (MPa)</td>
<td>88.6</td>
<td>0.91</td>
</tr>
<tr>
<td>Energy Density (MJ/m³)</td>
<td>30.6</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Table 7.5: Effect of Rate on Results at Failure for DSC Compound

As may be seen, extension rate, over this range, has no effect upon the measured properties of the DSC compound.
Table 7.6: Effect of Rate on Results at Failure for DNi Compound

Looking at the Nitrile compound, it can be seen that there is a slight difference between the different rates, with 300mm/min being lowest. However when looking at these results in combination with those for other compounds, the effect of rate of test, at least at the levels available, may be excluded for the purposes of this project.

c) Accuracy of Readings.

i) Uniaxial.

The uniaxial tests only had one main inaccuracy if care was taken in preparing the samples: the fitting of elastic constants. It was quickly found the six data points recorded (the five chosen extensions plus break) where insufficient to give an accurate fit for the elastic constants. Although this could be limited by careful choice of recorded extensions, more data points are really required to ensure an accurate representation.

ii) Biaxial.

While analysing the video test records, it was found that some measurements were more difficult to read than others, leading to inaccuracies. This was not due to the camera speed (25fps), picture resolution or quality of the equipment as a professional PAL camera, tape and recorder were used. Instead it was decided that the method of measurement was the cause, coupled with parallax.

The main problem was accurately reading the bubble position on the grid, especially locating the point of maximum width. When examined carefully these errors were found to be as follows:
Table 7.7: Errors in Initial Biaxial Readings.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Error (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>±2.5</td>
</tr>
<tr>
<td>Width</td>
<td>±2.5</td>
</tr>
<tr>
<td>Height at Max. Width</td>
<td>±5.0</td>
</tr>
</tbody>
</table>

As can be seen these errors become more significant at low readings, one reason why there are no data below 30mm height.

The other major problem with accuracy was with the pressure readings. Although the transducer had a fast sample rate and was accurate to a few millivolts, the voltmeter proved less so. When examining the test videos, it was found that a large increase in height occurred quickly with no indicated change in pressure. It was also noted that the voltmeter indicated the final pressure, or even increased pressure, a few frames after burst. After enquires with the manufacturer, it was discovered that although the display was accurate to ±1mV, it only sampled at 2Hz, too slow for this type of test. The improvements described in Chapter 8.2 overcame this problem.

However the computational simulation of the tests was found to be accurate for all except the measured pressure (see Section 7.3). The shape and dimensions of the inflated diaphragm matched the measurements, as did the simulated extensions. In order to assess the later, fiducial marks were placed on a few samples and the extensions calculated from their deflections. From these tests the simulation matched the measured values.

7.3 Material Representation Trials.

a) Initial Attempts to Calculate General Elastic Constants.

Owing to the nature of the biaxial test chosen (see Chapter 4.2), elastic constants were required to determine biaxial behaviour as stresses and extensions were not directly measurable. As uniaxial constants alone were found not to be accurate, a method of deriving general constants from uniaxial stress strain data and limited biaxial data was needed.
Investigations using a diaphragm simulation software (see Appendix A) with variable ratios of $C_{01}/C_{10}$ found that the pole extension ratio varied directly with Rivlin constant ratio. Also when equation 2.18 is applied to the equibiaxial load case, equation 7.1,

$$\sigma = 2\left(\lambda_1^2 - \frac{1}{\lambda_1^2}\right)\left|\frac{\partial W}{\partial l_1} + \lambda_1^2 \frac{\partial W}{\partial l_2}\right|$$

which for the Mooney-Rivlin model (equation 2.22 and 2.24) gives;

$$\sigma = 2\left(\lambda_1^2 - \frac{1}{\lambda_1^2}\right)\left|C_{10} + \lambda_1^2 C_{01}\right|$$

For this model it can be seen that for a given ratio of $C_{01}$ and $C_{10}$ the stress, and hence pressure (equation 5.10) increases linearly with their values.

Because of this it was hoped that a unique ratio of constants could be found for a given profile which could then be scaled to match the pressure. This was attempted by plotting extension ratios (calculated for a given height and width) against Rivlin constant ratio. As can be seen in Figure 7.1 these lines do not cross, indicating this method would not provide a solution.
Because of this, and the difficulty in getting an accurate fit with two constants up to the high extension required, it was decided not to proceed further but to work with the filament theory (Chapter 2.2 c) ii).

b) Application of Filament Theory.

i) Comparison of Measured and Calculated Pressures.

When the Filament constants where initially applied in “SIMDIA.BAS”, although the results where dimensionally correct the calculated pressures were not so precise. These calculated pressures showed large deviations from those measured for certain compounds. These pressures are summarised in Table 7.8 and Figure 7.2, the latter plotted at 0pphr carbon black when plotting supplied compounds of unknown formulation for completeness.
<table>
<thead>
<tr>
<th>Compound</th>
<th>Measured Burst Pressure (kPa)</th>
<th>Simulated Burst Pressure (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N0</td>
<td>73</td>
<td>220</td>
</tr>
<tr>
<td>N23</td>
<td>152</td>
<td>210</td>
</tr>
<tr>
<td>N33</td>
<td>250</td>
<td>265</td>
</tr>
<tr>
<td>N53</td>
<td>357</td>
<td>320</td>
</tr>
<tr>
<td>N26</td>
<td>173</td>
<td>210</td>
</tr>
<tr>
<td>N46</td>
<td>227</td>
<td>265</td>
</tr>
<tr>
<td>N66</td>
<td>316</td>
<td>300</td>
</tr>
<tr>
<td>S13</td>
<td>146</td>
<td>180</td>
</tr>
<tr>
<td>S23</td>
<td>176</td>
<td>105</td>
</tr>
<tr>
<td>S33</td>
<td>256</td>
<td>200</td>
</tr>
<tr>
<td>S53</td>
<td>391</td>
<td>300</td>
</tr>
<tr>
<td>S16</td>
<td>161</td>
<td>80</td>
</tr>
<tr>
<td>S26</td>
<td>179</td>
<td>120</td>
</tr>
<tr>
<td>S46</td>
<td>276</td>
<td>200</td>
</tr>
<tr>
<td>S66</td>
<td>382</td>
<td>300</td>
</tr>
<tr>
<td>ANB</td>
<td>221</td>
<td>220</td>
</tr>
<tr>
<td>AE</td>
<td>249</td>
<td>166</td>
</tr>
<tr>
<td>DNe</td>
<td>120</td>
<td>115</td>
</tr>
<tr>
<td>DNi</td>
<td>111</td>
<td>62</td>
</tr>
<tr>
<td>DSC</td>
<td>165</td>
<td>126</td>
</tr>
<tr>
<td>LIG</td>
<td>83</td>
<td>190</td>
</tr>
<tr>
<td>PU</td>
<td>175</td>
<td>360</td>
</tr>
<tr>
<td>P</td>
<td>144</td>
<td>160</td>
</tr>
</tbody>
</table>

Table 7.8: Comparison of Calculated and Measured Burst Pressures
Including the unknown compounds it can be seen that the results are evenly scattered about a 1:1 ratio of measured to predicted pressures (0% Δp). Statistical analysis of these results showed a strong correlation between the results and the ideal 1:1 line, which improved when the more elastic materials (NR gum, latex and polyurethane) were removed. With these unfilled compounds the error is likely to be caused by difficulty fitting the elastic constants over the whole extension range due to their extreme stress strain curves. With the other compounds though, as carbon black loading increases, the values get closer to the ideal value, with NR generally being closer than SBR.

ii) Correction of Filament Theory.

Because of the differences between the measured and simulated pressures, the filament theory of elasticity was modified to allow the measured and simulated pressures to be matched.
It is proposed that equation 2.34, for the tension in the elastic filament, is modified by a multiplying factor, $\beta$, which depends upon the difference, $\Delta \lambda$, between the two principal biaxial extension ratios. Thus:

$$T = \beta \left[ T_0 + A.\varepsilon + B.\varepsilon^n \right]$$  \hspace{1cm} -7.3

Moreover, in the absence of evidence to the contrary, it is assumed the factor $\beta$ varies linearly between the two extreme conditions, equibiaxial (equation 7.4) and uniaxial (equation 7.5):

$$\Delta \lambda = \lambda_1 - \lambda_2 = 0 \quad \text{and} \quad \beta = \beta_e$$  \hspace{1cm} -7.4

$$\Delta \lambda = \lambda_1 - \frac{1}{\sqrt[3]{\lambda_1}} \quad \text{and} \quad \beta = 1$$  \hspace{1cm} -7.5

This dependence is shown in Figure 7.3, where line (1) corresponds to a compound whose stiffness is enhanced biaxially, whereas line (2) applies to a compound whose stiffness is greater uniaxially.

![Figure 7.3: Variation of $\beta$ with $\Delta \lambda$.](image)

For some general condition $(\lambda_1, \lambda_2)$, the factor takes the value:
\[ \beta = \beta_e + \Delta \lambda \frac{1 - \beta_e}{\Delta \lambda_0} \]  

where \[ \Delta \lambda = |\lambda_1 - \lambda_2| \]

and \[ \Delta \lambda_0 = \lambda_1 - \frac{1}{\sqrt{\lambda_1}} \quad \text{(for } \lambda_1 > \lambda_2) \]

or \[ \Delta \lambda_0 = \lambda_2 - \frac{1}{\sqrt{\lambda_2}} \quad \text{(for } \lambda_2 > \lambda_1) \]

With \( \beta_e \) held at unity, the remaining parameters are obtained by fitting to the uniaxial data as mentioned above, whereby each parameter may be adjusted until a good fit is obtained. \( \beta_e \) can then be calculated using further software as described in further detail in Chapter 9. This program has indicated that \( \beta_e \) varies with extension ratio however.

Thomas\(^1\) has suggested that this requirement for a factor in the general biaxial case may be due to the theory being a function of \( I_1 \) only, as equation 2.35 can be represented by

\[ \varepsilon = \frac{\sqrt{I_1} - \sqrt{3}}{\sqrt{3}} \]  

Work to investigate this possible explanation has not been successful due to mathematical complexities.

iii) Use of Modified Filament Constants

Repeating the simulations, calculating the equibiaxial factor, \( \beta \), to match the measured pressures, it was found that \( \beta \) varied slightly with extension ratio as can be seen in Table 7.9.
<table>
<thead>
<tr>
<th>Rubber Compound</th>
<th>( \lambda = 3 )</th>
<th>( \lambda = 4 )</th>
<th>( \lambda = 5 )</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>N23</td>
<td>0.82</td>
<td>0.83</td>
<td>0.72</td>
<td>0.79</td>
</tr>
<tr>
<td>N33</td>
<td>0.85</td>
<td>0.83</td>
<td>0.81</td>
<td>0.83</td>
</tr>
<tr>
<td>N53</td>
<td>0.84</td>
<td>0.86</td>
<td>0.88</td>
<td>0.86</td>
</tr>
<tr>
<td>S23</td>
<td>-</td>
<td>1.04</td>
<td>1.18</td>
<td>1.11</td>
</tr>
<tr>
<td>S33</td>
<td>-</td>
<td>1.16</td>
<td>1.42</td>
<td>1.29</td>
</tr>
<tr>
<td>S53</td>
<td>1.35</td>
<td>1.50</td>
<td>1.64</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Table 7.9: Equibiaxial Multiplying Factors.

Neglecting the factors for S23 and S33 at the extension ratio of 3, where measured pressures were difficult to estimate and may be in error, the mean values for the three extension ratios were calculated. The simulation was then repeated using the mean values in Table 7.9. As can be seen in Table 7.10 and Figure 7.4, this matched the measured pressure well at all extension ratios, with the residual standard deviation falling from 39kPa (Figure 7.2) to 14kPa with all the data clustered around the one line.
<table>
<thead>
<tr>
<th>Rubber Compound ((\lambda) is pole extension ratio)</th>
<th>Measured</th>
<th>Simulated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(H) (mm)</td>
<td>(P) (kPa)</td>
</tr>
<tr>
<td>N23</td>
<td>(\lambda = 3)</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>(\lambda = 4)</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>(\lambda = 5)</td>
<td>75</td>
</tr>
<tr>
<td>N33</td>
<td>(\lambda = 3)</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>(\lambda = 4)</td>
<td>57</td>
</tr>
<tr>
<td></td>
<td>(\lambda = 5)</td>
<td>68</td>
</tr>
<tr>
<td>N53</td>
<td>(\lambda = 3)</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>(\lambda = 4)</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>(\lambda = 5)</td>
<td>63</td>
</tr>
<tr>
<td>S23</td>
<td>(\lambda = 3)</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>(\lambda = 4)</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td>(\lambda = 5)</td>
<td>72</td>
</tr>
<tr>
<td>S33</td>
<td>(\lambda = 3)</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>(\lambda = 4)</td>
<td>58</td>
</tr>
<tr>
<td></td>
<td>(\lambda = 5)</td>
<td>68</td>
</tr>
<tr>
<td>S53</td>
<td>(\lambda = 3)</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>(\lambda = 4)</td>
<td>57</td>
</tr>
<tr>
<td></td>
<td>(\lambda = 5)</td>
<td>66</td>
</tr>
</tbody>
</table>

Table 7.10: Measured and Simulated Heights and Pressures

(For approximate extension ratios of 3, 4 and 5)
Figure 7.4: Comparison of Simulated and Measured Pressures (Variable \( \beta \)).

It can also be seen in Table 7.10 that there is a slight difference between the simulated height for a given extension ratio, but not sufficient to cause concern. There was however difficulty calculating \( \beta \) for N0, LIG and PU.

Due to the large differences in pressure seen in Table 7.8, a very small \( \beta \) was required. However although an estimate could be extrapolated from pressures at higher \( \beta \)’s, “SIMDIA.BAS” could not calculate stresses and extensions for \( \beta \)’s less than 0.35. The \( \beta \)’s for the other industrial compounds could be successfully calculated however.
7.4 Initial Results.

a) Comparison of Tensile Strength Data with Available Results.

When the results of the commissioning tests were examined, it was suggested\(^2\) that the tensile test results for the initial tests were significantly lower than expected. Comparison with available results\(^3,4\) confirmed this observation, and later investigations, Chapter 9.6, showed this to be due to under cure.

b) Bubble Parameters.

On examination of the video records of inflation, it was found that the upper halves of the inflated diaphragms were, within the accuracy of the estimates, semi-ellipses. The semi-major axis, \(a\), is half the width and the semi-minor axis, \(b\), the difference between the pole height and the height to the point of maximum width. Moreover, irrespective of the height of a diaphragm and the compound from which it was moulded, the ratio of the minor axis to the major axis was constant. This is demonstrated by Figure 7.5, which includes the six compounds as well as data for latex and a polyurethane, all at three or four heights.

![Figure 7.5: Relationship Between Major and Minor Axes for the Top Ellipse](image)

The correlation coefficient is 0.981 with 54 degrees of freedom, indicating a significance approaching 100%. The ratio over the range \(a = 25\) to 70, given by the slope, is:
It was also found that there is a unique relationship between the heights and the widths of the diaphragms, Figure 7.6.

This is given, from $H = 30$ to $110$, by:

$$W = 22.7 + 0.882.H + 0.00211H^2$$

The correlation coefficient is 0.991, again with 54 degrees of freedom.

From these two equations it became possible to calculate the bubble shape just from a visual height measurement for the mixed compounds. Advantage of this was taken when the apparatus and experimental procedure were improved in order to simplify the measurements taken (see Chapter 8).

Comparison of this relationship with other published data proved difficult as many authors did not include height and width data. One such comparison, however, is possible with the data of Flint and Naunton\(^5\) for Latex, as shown in Table 7.11 for one height;
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Flint and Naunton (mm)</th>
<th>Predicted (from Height) (mm)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major Axis (a)</td>
<td>45.5</td>
<td>52.1</td>
<td>+15</td>
</tr>
<tr>
<td>Minor Axis (b)</td>
<td>38.6</td>
<td>46.4</td>
<td>+21</td>
</tr>
<tr>
<td>Height at Width</td>
<td>33.8</td>
<td>32.9</td>
<td>-3</td>
</tr>
</tbody>
</table>

Table 7.11: Comparison of Predicted and Measured Values

The predictions are not accurate but it should be noted that some of Flint & Naunton’s measurements were made using photography, and so may be prone to an unquantifiable parallax error (12% in our case) and an unknown error in readings from the grid (see Section 7.2 c).

c) Carbon Black Types.

When the uniaxial strength results for the two carbon black types were compared, it was surprising to find no statistical difference (less than 95% significance of differences), although the mean results for N660 were lower than those for N330 as expected. This was partly due to the large degree of scatter in the results.

Biaxially the significance of the difference between carbon black types was more pronounced, with statistical differences evident for most properties, one exception being engineering stress at break for SBR. This may indicate the biaxial test is more susceptible to reinforcement than the uniaxial test. Despite this, time constraints made it necessary to drop the N660 compounds from future tests.
d) **Gauge Extension Assumption.**

During the tensile tests it was decided to check the main assumption used when setting up the uniaxial fatigue machine. This assumption about the proportion of the extension that occurs in the gauge length was found to be in error, with large differences (up to 90%) in many cases. In order to cure this problem, new test method was developed as described in Chapter 9.3.

e) **Summary of Results.**

The results for these initial tests on the NR and SBR compounds are summarised in Figure 7.7 to Figure 7.9. In these figures, where no significant difference was found between the carbon black types only one curve is plotted. Both data sets are however plotted as points.
Chapter 7

Figure 7.7: Comparison of Extension Ratio at Break.

- NR Uniaxial
- NR Equi-Biaxial
- SBR Uniaxial
- SBR Equi-Biaxial N330
- SBR Equi-Biaxial N660
Figure 7.8: Comparison of Engineering Stress at Break.
Figure 7.9: Comparison of True Stress at Break.
In Figure 7.7 it can be seen that SBR is much less extensible than NR at low carbon black loadings when tensile tested. In the biaxial case however, the elongation at break of SBR increases significantly. This large difference in properties between uniaxial and biaxial load cases SBR can also be seen in Figure 7.8 and Figure 7.9 with a large increase in biaxial strength in SBR. Smaller increases can also be seen in NR. Figure 7.7 to Figure 7.9 indicate the expected weakness of SBR at low carbon black levels due the lack of crystallinity. The difference in SBR's properties in the biaxial load case at low carbon black levels was larger than expected and contradicts traditional opinion.

In all cases though, as black loading is increased, the differences between compounds and load cases is reduced. This suggests that the dominant reinforcing mechanism changes between test types. In a Uniaxial test stain crystallisation is dominant, as seen by the large difference between SBR and NR. As carbon black loading increases however strain crystallisation is hindered, reducing the difference between the two sets of results. In a biaxial test, these preliminary results suggest that carbon black reinforcement is dominant. This can be seen by the emergence of a significant difference between carbon black types with the biaxial case, and the large increase in SBR's properties.

Another explanation for the large increase in the strength of SBR in a biaxial test, and to a lesser extent NR's, is the lack of edge defects. As carbon black hinders crack growth, this is more apparent at low carbon black loadings. In NR, as strain crystallisation also hinders crack growth the change is smaller. In a biaxial sample however the flaws present are surface defects, or flaws within the rubber microstructure. As the stress distribution is also more even greater extensions can occur before significant crack growth.

These observations will be discussed more fully in Chapter 10.

f) Fatigue Tests.

At this stage of the project, the tests carried out using the fatigue apparatus were only for commissioning the equipment and developing a test methodology. Because of this only a few compounds were tested, including SBR, NR and latex. However despite the limited scope of the tests, some major deficiencies in the initial equipment design and test methods were highlighted.
With the uniaxial machine, it was soon found that the bottom clamps were too heavy, at around 180g, causing around 20mm of pre-stretch to occur in less stiff materials. It was also found that due to the difficulty in ensuring the samples were vertical, the clamps often “toggled” in either the up or down position, causing the failure detection system to fail.

Despite the lack of a significant difference between cut and moulded samples in the tensile test, a significant difference was found with fatigue. When the lives of cut and moulded dumbbells were compared, it could be seen, even without analysis, that moulded samples gave a marked reduction in scatter, and an increase in mean life. One example is S53 at 62.5% breaking extension where cut dumbbells had just 2% of the life of the moulded ones and almost twice the scatter. This difference was larger than expected, especially as cut dumbbells are the norm in industry. Because of the reduction in scatter it was decided that moulded dumbbells would be used in all future fatigue tests, and for consistency strength as well.

Problems were also encountered with the biaxial fatigue machine. It was noticed that, after a long test, the sensors were marking the top of the samples forming a circular cut over time. This was found to be due to a small O-ring placed on the sensor. This aimed to prevent damage to the sample by the metal sensor head and form an airtight seal when the rubber touches. In addition, when analysing one set of results, it was found that a life of several hundred cycles was recorded at a height greater than that required for burst on the strength test. This was attributed to the thickness of the bottom clamps, 10mm thick compared to the strength testers 5mm. This reduces the stress at the pole for a given height as more rubber is supported by the clamp ring.

The solutions to these problems are outlined in Chapter 8.
References

1 A G Thomas, *Private communication.*


4 Cabot carbon black data sheet standard results.

5 C F Flint and W J S Naunton, *Trans IRI, 12,* p367 (1937)

Chapter 8
Equipment Modifications.

8.1 Tensile Testing Apparatus.
Initially the Hounsfield tensile tester only output up to five selected stresses and the extension and stress at break. This proved insufficient for accurate results and so, after being approached, Hounsfield supplied additional software which allows up to 1000 pairs of stress and extension data to be saved. This required an update to the tensile analysis software, becoming “TNRHOUN.BAS” described in Appendix A and in Chapter 4.1.

To cater for this large increase in the number of points, the software now picks 25 evenly spaced points from each tensile test. The elastic constants are then fitted to a series of these sets of data, with a facility to ignore obviously bad tests.

8.2 Biaxial Strength Apparatus.
Due to problems in accuracy and speed of analysis outlined in Chapter 7.2 c), large modifications were required to the biaxial apparatus. Fortunately, the discovery of a fixed relationship between height and width during initial tests meant only height and pressure needed to be measured, the former by a laser distance device and the latter by a transducer.

To automate data collection, the output from these devices were logged by a computer via a fast analogue to digital converter (A to D) at intervals of 100ms, but with 1µs between height and pressure readings. The computer, via the “BXLAS.BAS” software described in Appendix A, then converted the voltage readings to height in mm and pressure in kPa, correcting for zero readings. A second software program, “BXANAL.BAS”, was then used to average the results in a second separate stage.

However initial tests showed that very little data was collected at the start of the test due to the air line pressure being applied too rapidly. However, replacement of the flow valve by a precision pressure regulator solved this problem. The valve also had the added advantage of allowing the pressure to be held at a set value to allow the height to settle. The final form of the apparatus is shown in Figure 8.1.
8.3 Uniaxial Fatigue Apparatus.

As described in Chapter 7.4 f), initial runs showed that the bottom clamps on the uniaxial fatigue apparatus were too heavy (≈180g), causing significant extensions (≈20mm) in low modulus compounds. After investigations with an NR latex compound using an equilibrium modulus tester which allowed the very small extensions at low loads to be measured, it was decided that a 50g mass gave an acceptable low extension of 5mm.

Initially, thinner aluminium versions of the original clamps in Figure 4.5 were tried, but although light, they were not suitable. First they were difficult to assemble as the original clamps had been hand fitted to the locations of the vertical guide posts. Second, they could stick in the “up” position at sample failure, resulting in a false reading. This was caused by a mixture of a “toggling” effect, caused by asymmetrical loading, and friction between the bushes and posts.

Because of this design failure, a clean sheet approach was tried with linear bearings instead of bushes and posts. These were attached to small lightweight aluminium clamps, with an overhanging support bracket to stop the tensile forces acting on the bearing, as can be seen in Figure 8.2.
Figure 8.2: New clamp design for Uniaxial Fatigue Apparatus.

This design was a success, with a mass of only 20g, and minimal friction to ensure correct operation of the failure detection system. It was however an expensive modification, costing 20% of the original purchase price.

8.4 Biaxial Fatigue Apparatus.

During the commissioning runs described in Chapter 7.4 f), two problems came to light. First, it was noticed that the thicker clamps on the biaxial fatigue machine were affecting the results of the fatigue tests. Hence it was decided to reduce the thickness of the clamps to those of the biaxial strength apparatus, 5mm. The second modification was undertaken to overcome marking of the samples by the height sensors which could initiate premature failures. In order to prevent this, lightweight aluminium flaps were added to the height adjustment bar. These were raised by the samples to touch the sensors, as shown in Figure 8.3.
In order to stop a flap dropping too far and damaging the surface of a sample, a tongue was fitted. Although light weight to limit any effect on the sample, it was soon found that the flaps were too light, as the air escaping from failed samples occasionally lifted them up to give false readings. In order to avoid this, a small mass (5g) was added to each flap and was found to cure the problem.

Figure 8.3: Location of Flaps on Biaxial Fatigue Apparatus.
Chapter 9.
Revised Test Methods.

9.1 Tensile Strength Tests.
Owing to the changes in equipment outlined in Chapter 8.1, the selection of the five extensions to be saved when setting up the tensile tester is less critical as the raw data can now be used. Saved at the end of the test, the raw data are fed into the “TNRHOUN.BAS” software, described in Appendix A. This outputs the required Filament constants and an average stress strain curve.

9.2 Biaxial Strength Tests.
Owing to the redesign of the apparatus the test method was changed substantially. No calibration or video recordings were required, although the latter was used to analyse a test more thoroughly when required. Instead, once the sample was clamped, its details were entered into the logging software and a continuously updated zero reading taken until the pressure was applied. Then the data were then logged until failure or the limit on the numbers of samples able to be stored is reached. The pressure was controlled by slowly increasing the setting of the precision regulator and, at intervals, was held until the height stabilised.

The raw pressure and height data from repeated tests were then fed into the “BIAXANL.BAS” software in order to produce an average height/pressure curve, as well as values at break. This average height pressure curve was then used at a datum for calculating stresses and β at different extension ratios (heights) to break using “SIMDIA.BAS” using the Filament constants for the particular compound. An example of the calculation of β is contained in Appendix A.4 d).

9.3 Uniaxial Fatigue Tests.
As the previous method of estimating the gauge extension was found to be inaccurate a new method has been employed. The sample was marked with two gauge marks 20mm apart, and when the appropriate throw has been set, the displacement at top
dead centre measured. This accurately gives the applied displacement, but requires the throw to be estimated.

The second modification concerns setting the speed. The constant 60Hz setting used originally gives variable clamp velocities for different extensions. To compensate for this, the speed was set to give a constant velocity (variable motor speed), based on the applied extension as follows:

\[ v = \omega \cdot r \]  

where \( v \) is the required velocity in mm/min, \( \omega \) the motor speed (rpm), and \( r \) the throw (mm). Although it was hoped to set this to the strength test speed of 500mm/min, this was impractical, and so 2000mm/min was used.

9.4 Biaxial Fatigue Tests.

No changes needed to be made.

9.5 Calculation of Elastic Constants.

a) Conversion of Filament Constants.

As the Filament theory is not presently supported by any commercially available FEA packages, a method is required to convert its parameters to an alternative form. Using the four constants, plus a relationship for the equibiaxial factor, pairs of stresses for any general biaxial load case as defined by the two extension ratios \( \lambda_1 \) and \( \lambda_2 \) can be calculated. If these states are chosen to cover a comprehensive range of deformations, analogous to conducting experiments using a biaxial stretching rig, the Rivlin constants may be fitted to the resulting data.

A maximum extension is first selected and, with this maximum extension, the program generates a sequence of combinations of the two extension ratios \( \lambda_1 \) and \( \lambda_2 \). Early in the research, six values of \( \lambda_1 \) were selected, equispaced between 1 (0% strain) and the maximum strain. For each value of \( \lambda_1 \), five values of \( \lambda_2 \) were selected as shown by Figure 9.1. It should be noted that equibiaxial conditions (\( \lambda_1 = \lambda_2 \)) are avoided for reasons related to the mathematics when fitting the Rivlin constants.
It soon became apparent that these combinations of the extension ratios gave too much weight to the lower values of $\lambda_1$ and $\lambda_2$. Consequently, the sequence was changed to that illustrated by Figure 9.2. Again six equispaced values of $\lambda_1$ are selected but the numbers of values of $\lambda_2$ are graduated from 5 at the lowest $\lambda_1$ to 10 at the highest.
For each of the 45 combinations of the two extension ratios, values of the filament theory parameters are used to calculate the principal true stresses, allowing for equibiaxial factor which is introduced to match uniaxial data to biaxial behaviour.

Now as noted before, if the Rivlin strain energy function, equation 2.17, is partially differentiated with respect to the two strain invariants ($I_1$ and $I_2$), expressions are obtained for the differences between any two of the three principal true stresses, as follows.

$$\sigma_i - \sigma_j = 2\left(\lambda_i^2 - \lambda_j^2\right)\left[\frac{\partial W}{\partial I_1} + \lambda_k^2 \frac{\partial W}{\partial I_2}\right]$$

where $i,j,k$ can each be 1,2,3.

For any combination of $\lambda_1$, $\lambda_2$ and $\lambda_3$ (the last from the assumed incompressibility), $\sigma_1$ and $\sigma_2$ have been calculated, whilst $\sigma_3 = 0$. Thus for any $\lambda_1$, $\lambda_2$ and $\lambda_3$, three stress differences are obtained, effectively increasing the number of data from 45 to 135. These data may then be subjected to multiple regression analysis, resulting in the Rivlin elastic constants $C_{ij}$.
For example, taking the Mooney-Rivlin expansion of the strain energy density function for simplicity (equation 2.22), and using the relationship given in equation 2.24, the equation 9.2 may then be rearranged as:

\[
\frac{\sigma_i - \sigma_j}{2(\lambda_1^2 - \lambda_2^2)} = C_{10} + \lambda_k^2 C_{01}
\]

which for higher order expansions also involve \((I_1 - 3)\) and \((I_2 - 3)\).

The last equation with \(i, j, k = 1, 2, 3\) is in the form of a regression equation: \(y = a + b \cdot x\); and it can now be seen why equibiaxial conditions must not be included among the combinations of \(\lambda_1\) and \(\lambda_2\), otherwise the LHS of the equation would result in division by zero.

b) Calculation of Elastic Constants.

The procedure for calculating general elastic constants is summarised in Figure 9.3.

The uniaxial test provides the key to the method as the uniaxial stress strain curve is used to calculate the filament coefficients. These, along with the measured values from the biaxial test, are then used in the simulation to calculate the equibiaxial factor \(\beta_e\) at different extension ratios. This, along with the filament constants, can then be used to calculate the elastic constants required for FEA.
9.6 Cure Determination.

a) Characterisation of Press Behaviour.

In order to investigate the cause of the possible under curing in the initial samples, as described in Chapter 7.4 a), the effect of different variables were investigated. These included investigating the temperature profile of the press platen, and the temperature distribution in the rubber during curing. Also the state of cure after different moulding times was investigated and compared with the curemeter results.

When the available presses were examined, it was found that one press had the best temperature control of the presses. This press had a variation of ±5°C over the platen, and so was used for subsequent sample preparation.

The temperature history of the rubber in the mould was investigated by inserting a thermocouple into the rubber during curing. This indicated that the rubber in the mould required a finite time to heat to temperature, unlike the near instantaneous rise in the cure meter. The recorded temperature profile is shown in Figure 9.4;

![Temperature profile of NR Gum in Typical Sheet Mould](image)

Figure 9.4: Temperature profile of NR Gum in Typical Sheet Mould.
The rise time varies slightly with polymer and the mould used, but Figure 9.4 is typical.

Based on this finding it was decided to look at how the state of cure varies with time in both the curemeter and mould, the latter being checked using tensile test data. A difference was found between the time taken in the mould for maximum tensile strength, taken as 100% cure, and the time suggested by the cure meter. However, efforts to quantify this difference as a simple function for different black loadings and polymers failed.

b) Characterisation of State of Cure.

Owing to the differences between the results from the curemeter and those obtained in the presses, it was decide to change the method used to determine state of cure from that outlined in Chapter 6.2.

To minimise temperature variations during moulding, it was decided to warm up the moulding press with the platens closed for at least one hour before use to ensure an even temperature distribution. The empty mould was then placed between the platens for 30 minutes before use.

A curemeter test during this time was used as an initial guide to the press times. Tensile sheets were then cured at various times either side of this guide figure and tensile tests carried out. The time for maximum cure was then specified as the time taken to give maximum tensile properties (stress at break.) This time was used for curing all further samples.

This method has been found to give good states of cure, and allows for the different temperature and cure properties of individual compounds.
Chapter 10
Strength Test Results.

10.1 Introduction.

In the next three Chapters, the test results are reported. In this Chapter only the strength tests are covered, with the fatigue and FEA tests being dealt with in later Chapters. Conducted after the improvements to the apparatus described in the previous chapters were completed, these results are in three sets. Sets 1 and 2 contain the results from two separate groups of experimentation on a range of compounds. These included 4 carbon black loadings of N330 in NR and SBR. The third set was to investigate the effect of mixing conditions and cure on physical properties. This uses a single formulation of SBR with 35pphr N330. The exact formulations are listed in Chapter 6.1

For clarity this Chapter splits these results into four main sections: 10.3 Uniaxial (Tensile) test results; 10.4 Elastic constants; 10.5 Biaxial test results; 10.6 Comparison and summary. In these sections the causes of the results are investigated, and possible differences between the uniaxial and biaxial results highlighted. To assist in this process the results were subjected to statistical analysis to check the significance of any changes found. Unfortunately however, owing to the nature of the biaxial test, no statistical comparison of biaxial and uniaxial results is possible.

10.2 State of Mix Checks.

a) Experimental Compounds.

When the state of mix was assessed for the general compounds, it was found during sample preparation that there were very few agglomerates present, indicating a good state of mix.

b) State of Mix Investigation Compounds.

In Table 10.1 the results of the thin section analysis on the variable mix compounds are given, along with the results for an ideal SBR compound with 35pphr N330.
Table 10.1: State of Mix Results.

Although the state of mix range of these compounds is not as broad as first planned they do give distinct results between the extremes for both strength and fatigue. Because of this only the moderate and excellent mix results will be used in future analysis.

10.3 Tensile Tests.

a) Tables of Results.

Table 10.2 through Table 10.4 list the tensile test results. However due to continuous improvements to the test methodologies, result set 1 (Table 10.2) does not contain data for strain energy density (SED).
Table 10.3: Uniaxial Test Results (Set 2).

<table>
<thead>
<tr>
<th>Black Loading (pphr)</th>
<th>λ (-)</th>
<th>σ_T (MPa)</th>
<th>SED (MJ/m^3)</th>
<th>300% σ_T (MPa)</th>
<th>λ (-)</th>
<th>σ_T (MPa)</th>
<th>SED (MJ/m^3)</th>
<th>300% σ_T (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>6.44</td>
<td>157.7</td>
<td>40.2</td>
<td>25.6</td>
<td>4.21</td>
<td>46.4</td>
<td>18.0</td>
<td>40.6</td>
</tr>
<tr>
<td>35</td>
<td>4.56</td>
<td>95.2</td>
<td>30.4</td>
<td>64.4</td>
<td>5.81</td>
<td>107.5</td>
<td>33.3</td>
<td>33.2</td>
</tr>
<tr>
<td>50</td>
<td>5.98</td>
<td>181.0</td>
<td>72.2</td>
<td>73.4</td>
<td>6.91</td>
<td>186.9</td>
<td>74.2</td>
<td>47.0</td>
</tr>
</tbody>
</table>

Table 10.4: Uniaxial Test State of Mix Results.

<table>
<thead>
<tr>
<th>Cure (%)</th>
<th>λ (-)</th>
<th>σ_T (MPa)</th>
<th>SED (MJ/m^3)</th>
<th>300% σ_T (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mod.</td>
<td>excel.</td>
<td>mod.</td>
<td>excel.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>7.86</td>
<td>8.93</td>
<td>135.5</td>
<td>138.96</td>
</tr>
<tr>
<td>100</td>
<td>5.93</td>
<td>5.81</td>
<td>101.46</td>
<td>107.52</td>
</tr>
<tr>
<td>200</td>
<td>4.30</td>
<td>4.64</td>
<td>76.12</td>
<td>76.79</td>
</tr>
</tbody>
</table>

b) Statistical Analysis.

In this section, and Section 10.5b), the set 2 results are analysed statistically. For this analysis a normal distribution has been assumed. As in Chapter 7 for a difference to be considered significant, the significance of differences must be at least greater than 95%, and preferably greater than 97.5%.

i) Uniaxial Engineering Stress at Break.

Neglecting any datum obviously in error, the means of all six compounds were compared in a single statistical analysis. It showed the differences are approaching 100% significant. They were then compared in smaller groups to determine the significances of individual differences between selected compounds. It was found there are no differences between the three compounds: N23, N33 and S33. The results are summarised in Table 10.5.
### Table 10.5: Uniaxial Engineering Stress at Break.

<table>
<thead>
<tr>
<th>Compound</th>
<th>Number of Samples</th>
<th>Mean Value (MPa)</th>
<th>Standard Deviation (MPa)</th>
<th>Percentage Std.Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>N23</td>
<td>2</td>
<td>22.5</td>
<td>0.07</td>
<td>0.3%</td>
</tr>
<tr>
<td>N33</td>
<td>4</td>
<td>21.0</td>
<td>2.12</td>
<td>10.1%</td>
</tr>
<tr>
<td>N53</td>
<td>4</td>
<td>30.3</td>
<td>1.57</td>
<td>5.2%</td>
</tr>
<tr>
<td>S23</td>
<td>3</td>
<td>12.2</td>
<td>2.76</td>
<td>22.6%</td>
</tr>
<tr>
<td>S33</td>
<td>6</td>
<td>19.0</td>
<td>1.30</td>
<td>6.8%</td>
</tr>
<tr>
<td>S53</td>
<td>7</td>
<td>26.9</td>
<td>0.66</td>
<td>2.5%</td>
</tr>
</tbody>
</table>

### ii) Uniaxial Extension Ratio at Break.

Using the same statistical procedure, all the extension ratio differences are highly significant apart from that between N23 and N53: Table 10.6

<table>
<thead>
<tr>
<th>Compound</th>
<th>Number of Samples</th>
<th>Mean Value</th>
<th>Standard Deviation</th>
<th>Percentage Std.Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>N23</td>
<td>2</td>
<td>6.11</td>
<td>0.08</td>
<td>1.3%</td>
</tr>
<tr>
<td>N33</td>
<td>4</td>
<td>4.58</td>
<td>0.27</td>
<td>5.8%</td>
</tr>
<tr>
<td>N53</td>
<td>4</td>
<td>6.03</td>
<td>0.30</td>
<td>4.9%</td>
</tr>
<tr>
<td>S23</td>
<td>4</td>
<td>4.22</td>
<td>0.53</td>
<td>12.7%</td>
</tr>
<tr>
<td>S33</td>
<td>6</td>
<td>5.66</td>
<td>0.30</td>
<td>5.3%</td>
</tr>
<tr>
<td>S53</td>
<td>7</td>
<td>6.67</td>
<td>0.43</td>
<td>6.4%</td>
</tr>
</tbody>
</table>

### Table 10.6: Uniaxial Extension Ratio at Break.

### iii) Uniaxial True Stress at Break.

When analysed, there are very significant differences between the three carbon black loadings in each of the two polymers: Table 10.7. However, the following two pairs of compounds are not significantly different: [N33, S33] and [N53, S53].

Page 100
iv) Uniaxial 300% “Modulus”.

Owing to its common usage in rubber technology, the data for the 300% “modulus” were analysed. In fact the data refer to the engineering stress at 300% extension.

There are no significant differences between the three compounds: [N23, S23, S33], due in part to the high scatter in the data for the latter two compounds: Table 10.8.
c) Discussion of Results.

i) Experimental Compounds

The uniaxial test results were found to be very scattered for all the tests undertaken. This degree of scatter can be seen in some of the results by the lack of a significant difference between results where one would be expected, such as N330 and N660 in the initial tests. This degree of scatter was however reduced with the introduction of moulded dumbbells, and by careful attention to procedure, but was partly caused by the nature of the test.

With the apparatus used, especially the grips, some degree of slippage was present, probably causing an uneven stress distribution. It was also difficult to get the samples vertical and the gauge marks aligned. Friction damage as the sample slid over the clamp during extension was also likely on the neck. Because of this failure in these samples often occurred near the neck and very rarely between the gauge marks as required by BS903\(^1\). Samples failing on the neck were discarded, but if every sample failing outside the marked gauge was also neglected as suggested, an impractically large number of samples would have been required. All these problems will have contributed to the scatter in the results.

In order to simplify analysis, the results from set 1 and 2 have been combined by plotting a best fit through both sets of data in Table 10.9. This allows for the removal of possible "bad" compounds such as S23 in set 1 and N33 in set 2 which have results which appear too low in comparison to the other results.

<table>
<thead>
<tr>
<th>Black Loading (pphr)</th>
<th>NR</th>
<th>SBR</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>( \sigma_T ) (MPa)</td>
<td>300% ( \sigma_T ) (MPa)</td>
</tr>
<tr>
<td>0_{min}, 10_{max}</td>
<td>5.64</td>
<td>90.5</td>
</tr>
<tr>
<td>20</td>
<td>6.24</td>
<td>159.1</td>
</tr>
<tr>
<td>35</td>
<td>5.17</td>
<td>126.7</td>
</tr>
<tr>
<td>50</td>
<td>5.00</td>
<td>103.4</td>
</tr>
</tbody>
</table>

Table 10.9: Combined Uniaxial Test Results.
Examining the combined results, it can be seen that the strength of SBR is less than NR at low carbon black loadings as expected. However above 35pphr the statistical analysis suggests very little difference between the two results, as can be seen in Figure 10.2. It is also interesting to note that the increase in stress and extension at break for SBR with carbon black loading is almost linear, indicating the increase in strength is due to only carbon black reinforcement. With NR on the other hand, the effect of crystallinity and its
interaction with carbon black reinforcement can be seen. This shows as the initial rise in properties with increased carbon black as crystallinity and carbon black reinforcement work together, followed by a decrease as the carbon black hinders the formation of a crystalline structure. This interaction is seen most clearly in the true stress at 300% extension Figure 10.3;

![Figure 10.3: Uniaxial True Stress at 300%](image)

The low value of SBR at low carbon black loadings is probably caused by edge defects as gum SBR is very susceptible to crack initiation from edge defects. The addition of carbon black however hinders crack growth, increasing strength. NR on the other hand as reduced crack growth from the start due to its crystalline nature.

ii) State of Mix Compounds.

Looking at the uniaxial data it can be seen that increasing state of mix causes a slight reduction in strength and stiffness, as can be seen in Figure 10.4 for three levels of cure;
This is attributed to both the increased effective reinforcement (see Table 10.1) present in the less well mixed samples due to immobilised rubber in the larger carbon black particles, and to damage to the elastomer with extended mixing. However at very low states of mix, the literature suggests the tensile properties will be reduced\(^2\).

Cure affects the properties as expected with a peak true stress at break at 100% cure as expected, and a decrease in extension at break with increasing cure, the latter due to cross link density. The increase in 300% "modulus" with cure may point to a reason for the initial undercuring of samples (see Chapter 7.4a)). The Wallace cure meter uses maximum torque to define 100% cure. As torque is a stiffness (modulus) measure, and stress is a strength measure, this may explain part of the difference in times indicated between the two methods, as the cure time for maximum stiffness is likely to be different than that required for maximum strength.

10.4 Elastic Constants

a) Filament Constants.

In the process of fitting elastic constants to the uniaxial data, it was noticed that there may be many possible fits for a given set of tensile data. These differences have not been found
to significantly alter the final results, although there may be differences if the results are examined closely. Two such fits are shown in Figure 10.5 and Figure 10.6 (screen dumps from "TNRHOUN.BAS");

Figure 10.5: Filament Constant Fit to Uniaxial Data.
(N53, Constants from Table 10.10)
Figure 10.6: Alternative Filament Constant Fit to Uniaxial Data.

(N53, T = 2.3 + 8.5ε + 3ε^{1.2})

This variation is due to differing degrees of fit at different points over the whole strain range. Generally it is easy to get an acceptable fit over a particular range of extensions, but more difficult to get a good fit over the whole extension range to break. Variations between users is also likely due to the expectations of the user, such as the degree of fit required, and the extension range of interest.

The differences between fits are partly due to the present method of fitting. If a more automated approach was used such discrepancies would be limited. However such a procedure would be mathematically complex as different log scales are required at different extensions to get a good fit. A more general power series has also been tried to solve this problem, equation 10.1, but the fit also varied from user to user, as well as being more complicated to adjust to improve the level of fit.
Table 10.10 lists the Filament constants used in the biaxial simulations.

<table>
<thead>
<tr>
<th>Compound</th>
<th>Pretension $(T_0)$</th>
<th>Linear Coef. $(A)$</th>
<th>Power Coef. $(B)$</th>
<th>Power Index $(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N23</td>
<td>1.80</td>
<td>0.492</td>
<td>2.936</td>
<td>2.195</td>
</tr>
<tr>
<td>N33</td>
<td>2.50</td>
<td>0.010</td>
<td>9.000</td>
<td>1.313</td>
</tr>
<tr>
<td>N53</td>
<td>2.50</td>
<td>0.000</td>
<td>13.394</td>
<td>0.985</td>
</tr>
<tr>
<td>S23</td>
<td>1.88</td>
<td>0.025</td>
<td>2.531</td>
<td>1.875</td>
</tr>
<tr>
<td>S33</td>
<td>1.64</td>
<td>0.000</td>
<td>3.808</td>
<td>1.512</td>
</tr>
<tr>
<td>S53</td>
<td>2.90</td>
<td>0.000</td>
<td>4.400</td>
<td>1.400</td>
</tr>
</tbody>
</table>

Table 10.10: Filament Constants (Set 2).

If a fit is chosen that is an acceptable, rather than precise, fit over the whole extension range, such as the data in Table 10.10, some trends are apparent. As carbon black content is increased the pretension $T_0$ and power coefficient $B$ also increase, consistent with theory and previous fits. The pretension $T_0$, or the initial strength of the material, would be expected to rise with carbon black loading. The power coefficient would also be expected to rise as the magnitude of stress for a given extension increases with carbon black loading. Conversely as carbon black loading increases the gradient of the final rise would be expected to decrease, hence the reduction in power coefficient.

Despite these problems in fitting the constants over the whole test range, the Filament constants do provide an acceptable fit. This can be seen if the measured and calculated results for 300% modulus are compared.


### Table 10.11: Comparison of Measured and Calculated Uniaxial 300% Modulus.

<table>
<thead>
<tr>
<th>Compound</th>
<th>Calculated &quot;Modulus&quot; (MPa)</th>
<th>Measured &quot;Modulus&quot; (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N23</td>
<td>7.84</td>
<td>7.1</td>
</tr>
<tr>
<td>N33</td>
<td>15.31</td>
<td>16.1</td>
</tr>
<tr>
<td>N53</td>
<td>19.81</td>
<td>18.3</td>
</tr>
<tr>
<td>S23</td>
<td>6.13</td>
<td>10.2</td>
</tr>
<tr>
<td>S33</td>
<td>6.61</td>
<td>8.4</td>
</tr>
<tr>
<td>S53</td>
<td>9.27</td>
<td>12.6</td>
</tr>
</tbody>
</table>

b) Equibiaxial Factor.

During calculation of the required equibiaxial factors, it was noticed that different values were required at different extensions in order to match the pressures, as shown in Figure 10.7.

![Figure 10.7: Variation of \( \beta \) with Extension Ratio (Set 2).](image-url)
The variation in $\beta$ seen in Figure 10.7 was not originally foreseen, although in some cases there is very little difference in $\beta$ with extension. However, the final (highest extension) result, is in general agreement with the original hypothesis in Chapter 7.3 b) with $\beta$ approaching unity with increased carbon black loading. The individual values though deviate with reducing extension, with the value of $\beta$ for both polymers becoming less than 1, though $\beta$ generally increases with extension. This is possibly related to the stress extension curve as the gradient increases with extension due to the increasing effects of carbon black reinforcement or crystallinity. As these effects are suspected to be the reason for the need for an equibiaxial factor, the value of $\beta$ alters.

There is no apparent trend however with carbon black loading although this is likely to be due to only one set of data being compiled with variable $\beta$. Ignoring N33, which is suspected of being “bad” in set 2 and S33 as only one value of $\beta$ is available, the average values in Table 10.10 do appear to have a trend.

<table>
<thead>
<tr>
<th>Compound</th>
<th>Equibiaxial Factor ($\beta$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N23</td>
<td>0.90</td>
</tr>
<tr>
<td>N33</td>
<td>0.69</td>
</tr>
<tr>
<td>N53</td>
<td>0.81</td>
</tr>
<tr>
<td>S23</td>
<td>0.74</td>
</tr>
<tr>
<td>S33</td>
<td>1.55(^{\dagger})</td>
</tr>
<tr>
<td>S53</td>
<td>1.09</td>
</tr>
</tbody>
</table>

Table 10.12: Equibiaxial Factor (Set 2).

\(^{\dagger}\)Single value of $\beta$ only for this compound.

In NR the values decrease, moving away from unity, whereas in SBR they approach unity. The trend towards unity is expected from the original hypothesis as carbon black loading reduces the differences in results between uniaxial and biaxial results. However both SBR and NR do not completely follow the original idea, with SBR having a value less than unity, and NR moving away from unity.
Despite these differences to the expected trend, with the value of $\beta$ included, a plot of reduced stress (see Figure 10.8) can be plotted. However, in order to produce this plot, the equibiaxial extensions need to be converted to compression data. According to Bhate et al.\(^3\),

\[
\lambda_c = \frac{1}{\lambda_B^2}
\]

\[
\sigma_c = -\sigma_B
\]

where the subscripts B and C relate to equibiaxial extension and uniaxial compression respectively.

![Graph showing reduced stress against 1/\(\lambda\)](image)

**Figure 10.8: Plot of Reduced Stress Against $\lambda$**

Although comparisons at low extensions are not possible due to the lack of data, it can be seen that the Filament constants give a reasonable fit in both uniaxial tension and compression, unlike the example Mooney-Rivlin plot in Figure 2.3. The slight difference between experimental and theoretical results in compression at low strains is probably due to a fixed $\beta$ being used to produce the theoretical data.
c) Summary.

It appears that Filament constants can be used to accurately represent the multiaxial behaviour of rubber compounds with the inclusion of a equibiaxial factor. However more work is required to investigate the detailed mathematical relationship between the filament theory and $I_1$. The effect of different quantities on the equibiaxial factor also requires further investigation, including its value at intermediate general biaxial conditions.

10.5 Biaxial Tests.

a) Tables of Results.

<table>
<thead>
<tr>
<th>Black Loading (pphr)</th>
<th>$\lambda$ (-)</th>
<th>NR $\sigma_T$ (MPa)</th>
<th>300% $\sigma_T$ (MPa)</th>
<th>SBR $\sigma_T$ (MPa)</th>
<th>300% $\sigma_T$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (SBR)</td>
<td>6.7</td>
<td>134.7</td>
<td>15.6</td>
<td>6.4</td>
<td>165.6</td>
</tr>
<tr>
<td>20</td>
<td>5.6</td>
<td>139.8</td>
<td>39.3</td>
<td>5.3</td>
<td>127.8</td>
</tr>
<tr>
<td>35</td>
<td>4.9</td>
<td>118.8</td>
<td>68.6</td>
<td>6.7</td>
<td>190.4</td>
</tr>
<tr>
<td>50</td>
<td>5.1</td>
<td>143.0</td>
<td>83.4</td>
<td>4.9</td>
<td>119.3</td>
</tr>
</tbody>
</table>

Table 10.13: Biaxial Test Results (Set 1).

<table>
<thead>
<tr>
<th>Black Loading (pphr)</th>
<th>$\lambda$ (-)</th>
<th>NR $\sigma_T$ (MPa)</th>
<th>SED (MJ/m³)</th>
<th>300% $\sigma_T$ (MPa)</th>
<th>SBR SED (MJ/m³)</th>
<th>300% $\sigma_T$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5.30</td>
<td>98.3</td>
<td>52.6</td>
<td>41.2</td>
<td>153.2</td>
<td>108.8</td>
</tr>
<tr>
<td>35</td>
<td>5.10</td>
<td>124.8</td>
<td>82.6</td>
<td>68.0</td>
<td>113.4</td>
<td>67.2</td>
</tr>
<tr>
<td>50</td>
<td>4.81</td>
<td>117.1</td>
<td>90.5</td>
<td>75.2</td>
<td>83.0</td>
<td>124.0</td>
</tr>
</tbody>
</table>

Table 10.14: Biaxial Test Results (Set 2).
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Table 10.15: Biaxial State of Mix Test Results.

<table>
<thead>
<tr>
<th>Cure (%)</th>
<th>$\lambda$ (%)</th>
<th>$\sigma_T$ (MPa)</th>
<th>SED (MJ/m$^3$)</th>
<th>300% $\sigma_T$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mod.</td>
<td>excel.</td>
<td>mod.</td>
<td>excel.</td>
</tr>
<tr>
<td>65</td>
<td>4.95</td>
<td>3.81</td>
<td>109.20</td>
<td>67.25</td>
</tr>
<tr>
<td>100</td>
<td>4.91</td>
<td>4.57</td>
<td>141.65</td>
<td>113.44</td>
</tr>
<tr>
<td>200</td>
<td>4.82</td>
<td>4.48</td>
<td>137.10</td>
<td>110.65</td>
</tr>
</tbody>
</table>

Table 10.15: Biaxial State of Mix Test Results.

b) Statistical Analysis.

In the biaxial test the only measurements taken on repeated samples were the height of the diaphragm and the internal pressure at failure. Consequently, only these data for the six compounds can be compared.

i) Diaphragm Height at Burst.

After neglecting one or two data from each compound because they were obviously in error, the means of all six compounds were then compared in a single statistical analysis, which showed the differences are approaching 100% significant. They were then compared in smaller groups to determine the significances of individual differences between selected compounds. The results are summarised in Table 10.16.

<table>
<thead>
<tr>
<th>Compound</th>
<th>Number of Samples</th>
<th>Mean Value (mm)</th>
<th>Standard Deviation (mm)</th>
<th>Percentage Std.Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>N23</td>
<td>5</td>
<td>79.8</td>
<td>2.7</td>
<td>3.3%</td>
</tr>
<tr>
<td>N33</td>
<td>4</td>
<td>68.9</td>
<td>2.3</td>
<td>3.3%</td>
</tr>
<tr>
<td>N53</td>
<td>6</td>
<td>62.0</td>
<td>1.5</td>
<td>2.5%</td>
</tr>
<tr>
<td>S23</td>
<td>3</td>
<td>77.3</td>
<td>1.8</td>
<td>2.3%</td>
</tr>
<tr>
<td>S33</td>
<td>6</td>
<td>76.7</td>
<td>5.1</td>
<td>6.7%</td>
</tr>
<tr>
<td>S53</td>
<td>6</td>
<td>68.3</td>
<td>5.7</td>
<td>8.4%</td>
</tr>
</tbody>
</table>

Table 10.16: Diaphragm Height at Burst.
In more detail, the three NR compounds all differ significantly in height from each other (>99.9%) but only S53 differs from the other two SBR compounds (96.2% and 97.6%).

Comparing the polymers at equal carbon black loading, the 35pphr (97.7%) and 50pphr (97.3%) compounds are significantly different but not the 20pphr compounds (79.5%).

ii) Diaphragm Pressure at Burst.

The burst pressures for the six compounds were more self-consistent and there was no reason to neglect any of the data. As before, the means of all the compounds were compared in a single analysis and, overall, the significance of the differences approaches 100%. The results are summarised in Table 10.17.

<table>
<thead>
<tr>
<th>Compound</th>
<th>Number of Samples</th>
<th>Mean Value (kPa)</th>
<th>Standard Deviation (kPa)</th>
<th>Percentage Std.Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>N23</td>
<td>7</td>
<td>126</td>
<td>10.6</td>
<td>8.4%</td>
</tr>
<tr>
<td>N33</td>
<td>5</td>
<td>213</td>
<td>7.6</td>
<td>3.6%</td>
</tr>
<tr>
<td>N53</td>
<td>6</td>
<td>244</td>
<td>12.3</td>
<td>5.0%</td>
</tr>
<tr>
<td>S23</td>
<td>4</td>
<td>166</td>
<td>16.3</td>
<td>9.8%</td>
</tr>
<tr>
<td>S33</td>
<td>6</td>
<td>211</td>
<td>13.4</td>
<td>6.4%</td>
</tr>
<tr>
<td>S53</td>
<td>8</td>
<td>225</td>
<td>16.3</td>
<td>7.2%</td>
</tr>
</tbody>
</table>

Table 10.17: Diaphragm Pressures at Burst.

The three NR compounds differ significantly from each other (99.9%); but the burst pressures of S33 and S53 do not (88.9%), although they differ from S23 (99.9% and >99.9%).

Comparing the polymers at equal carbon black loading, the 20pphr (99.9%) and 50pphr (96.4%) compounds differ but not the 35pphr compounds (20.2%).

From Table 10.16 and Table 10.17, it may be seen that all the compounds differ one from the other either in diaphragm height or internal pressure.
c) Mode of Failure.

During the biaxial tests, it quickly became apparent that the samples from different families of compounds failed in a distinct manner. In all bar highly elastic materials such as NR gum, and polyurethane, failure started at the pole, the point of highest stress, and proceeded radially to the clamp. The pattern of failure was however dependant on the polymer under test, and to a lesser degree carbon black loading.

i) Failure Patterns.

The failure patterns fell into two main types, “clover leaf” and “radial”. “Clover leaf” failure occurs predominantly in SBR and high SBR content blends and can be seen for SBR in Figure 10.9. The radial failure pattern, Figure 10.10, generally occurs in NR, and the Nitrile compound tested. Flint and Naunton\(^4\) also suggest Latex fails in a radial manner, although latex tests on this apparatus usually did not.
Figure 10.9: "Clover leaf" Failure Pattern.
Figure 10.10: Radial Failure Pattern.
In both types of failure, increased carbon black loading was found to increase the number of “petals” present, as can be seen in the previous Figures. Also the amount of rubber missing from the sample can be seen to reduce with carbon black loading.

There is also evidence that the type of failure occurring may be rate dependant as it is difficult to control the rate exactly, even with the regulator. When a latex sample was being tested, too large a pressure was applied at the start of the test, causing the sample to fail in a highly radial manner. This is opposed to the more usual circumferential pattern with this apparatus.

ii) Probable Causes of Failure.

The cause of these failure modes has been examined in the past by Flint and Naunton and Treloar. Flint and Naunton suggest that as the sample is inflated, the molecules align radially and, in crystalline materials, crystallise. Failure then starts at the pole, and progresses radially along the aligned crosslinks which are usually weaker than the rubber molecules. This links with their observation of increasing numbers of petals with cure which as can be seen in Figure 10.11 is the same as the tests reported here.

![Figure 10.11: Effect of Cure on Failure Pattern (S33).](image)
Treloar discounts this theory as radial failure occurs in both crystallising and non-crystallising materials. This work also found radial failure in non-crystallising-polymers such as Nitrile which was also found to be strongly radial. Treloar also suggests the degree of orientation exhibited (defined by $\lambda_R / \lambda_C$ according to Treloar where $\lambda_R$ and $\lambda_C$ are the radial and circumferential extension ratios respectively) was not sufficient for such a degree of orientation to occur. Instead Treloar suggests that failure starts as a hole in the pole, which reduces $T_R$, and increases $T_C$ (the radial and circumferential tensions respectively) at the pole. This starts a crack which then propagates, after which multiple cracks are thought to start as the circumferential tension in the hole continues to increase. This is shown in Figure 10.12.

![Figure 10.12: Treloar's Suggested Crack Initiation.](image)

However, during the series of tests reported here, it was noticed that just prior to failure the sample was heard to creak, and if the pressure was held constant, the height would continue to increase to burst. It was also observed during a test on an EPDM sample that the surface began to peel back just prior to failure. This delamination is also apparent when failed samples are observed, as can be seen in Figure 10.13.
Figure 10.13: Micrograph of Failure Surface (6.5x magnification, N53).

It was also observed when examining the fracture surfaces that they often were fibrous, seen in Figure 10.14;

Figure 10.14: Micrograph of Failure Surface (40x magnification, N53).
From these observations it would appear that failure in biaxial samples is caused by delamination. The top surface fails first, either due to increased cure\textsuperscript{6} due to closer proximity to the mould, or increased stress from being on the outside surface, although the latter is likely to be marginal. This outer surface failure causes increased loading in the inner surface, and also weakens the material, hence the reduced pressure required for continued inflation. The tearing back of the top surface may also be indicated by the fibrous nature of the initial failure zone. This draws analogies to the observations of Goldburg et al\textsuperscript{7} of bundles of fibres forming at the crack tip of tensile tear specimens, albeit macro rather than microscopic.

Also apparent in some samples are distinct ridges along the edges of the petals of some samples (Figure 10.15).

![Figure 10.15: Ridge Pattern on Failed Biaxial Sample (N33).](image)

This pattern was symmetrical across the width of the sample, although the teeth were opposed as can be seen.
Another interesting feature was that the pattern was consistent across a range of samples regardless of polymer or carbon black content, allowing two "petals" from different samples to mesh, as can be seen in Figure 10.17.
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Being non compound specific, and intermittent within samples as well as compounds, the cause could be many things. It is possible that under certain circumstances the crack is exhibiting an extreme form of knotty tear. Such tearing is not expected to be quite so regular though. A molecular bundling cause as suggested by Goldburg is also unlikely, again due to the regularity of the pattern. However as the pattern does not appear to be affected by compound, it is likely to be due to a load distribution caused by the test, rather than a molecular cause.

This cause is also suggested by other authors who have noticed similar patterns during tests on rubber compounds. Thomas, while investigating the failure of inflated latex spheres, found similar patterns on the tear edges. Although unsure of the exact cause of these patterns, tear tests on uniaxial, pure shear and equibiaxial samples indicated that such patterns are unique to an equibiaxial sample. This he attributed to the path of reducing energy being equal in both directions which may cause the fracture path to weave.

Andrews also noticed wave like fracture patterns when investigating the brittle fracture of NR compounds below their glass transition temperature. These patterns he attributed to the interaction of the stress and fracture waves as they pass through the sample, the former leading the latter. Although conducted at cryogenic temperatures, the Youngs modulii of polymers at fracture are closer to glassy polymers than elastomers in their normal state, making this a likely reason for the patterns described in this section. Why it should only happen during an equibiaxial test is uncertain.

Without further work the exact causes of these failure patterns is uncertain, and further investigation may reveal more information about the molecular behaviour of rubber compounds under multiaxial loading.

d) Discussion of Results.

Owing to the multi stage nature of the biaxial result analysis, analysis of the scatter is difficult as only height and pressure are measured directly. Analysis of these results however does indicate that the results are more repeatable than their uniaxial equivalents. This is possibly due to the more simple clamping, and the probability that the strength of the compound is being measured rather than the resistance to initial edge defects.
With the biaxial test though some degree of slippage does occur, evident by marks on the sample where clamping occurs. Normally the clamping mark is less than 1mm thick, however examinations of latex samples, one of the hardest materials to clamp successfully, indicates a small degree of slippage. When measured, the maximum clamp mark on failed samples was only 2 to 4mm larger in diameter than the clamp diameter. If this increase is compared to the overall circumference of an inflated diaphragm, assuming a spherical profile for simplicity, at a height of 100mm (less than break for latex), it is less than 1%.

Another cause of error is the failure location. In uniaxial tests it is obvious if failure has occurred outside the gauge length. However, unless the failure occurs well away from the pole it is less obvious where biaxial failure has been initiated. Examinations of failed samples indicate that failure occurs usually within 2 or 3mm of the centre of the uninflated sample. Although only general biaxial conditions exist away from the pole, such a small deviation is unlikely to cause a large change from equibiaxial conditions.

The nature and location of the test did however cause some problems. Being in an environment surrounded by 3 phase electrical equipment caused problems with interference on the data signal. Although this was partially filtered electronically, it required the sampling rate of the laser to be reduced to get the required 0.5mm accuracy. Another problem was the laser not getting a good reflection of the rubber surface, although the application of a small amount of aluminium powder to the sample surface at the centre of the sample solved this problem.

The main concern however was the number of steps required to get the stress strain data. With each stage prone to error, the errors unless care was taken could accumulate. Some of these errors could be reduced by improvements to the apparatus, and by more automation of the curve fitting process in “BXANAL.BAS” as well as “TNRHOUN.BAS”. Such alterations are described in Chapter 13.2.

Despite these problems checks at the early stages of the project indicate the results are valid. In these checks some samples were marked and direct measurements taken, as described in Chapter 7.2 c), with close correlation to the simulated results. The “SIMDIA.BAS” software was also based on mathematics proven in previous unpublished
baxial tests\textsuperscript{10} which had more comprehensive measurements to check against. The results also compare well to previous published\textsuperscript{11} and unpublished data\textsuperscript{10}, both of which employed fiducial lines on the sample.

![Graph of Extension Ratio vs Black Loading (pphr)]

**Figure 10.18:** Comparison of Published and Current Data (Extension Ratio).

The data for the Figure are for SBR with N330 with a sulphur cure, although the exact formulations vary. It can be seen that good correlation exists with extension ratio at break for all three sets. However with true stress there is a large difference between the previous data with the current data fitting between the two. This makes a graphical comparison of little value, although it may be indicative of an increased sensitivity in the biaxial test to compounding as the uniaxial values are similar.

In Table 10.18 both sets of results are combined.
Table 10.18: Combined Biaxial Test Results.

<table>
<thead>
<tr>
<th>Black</th>
<th>NR</th>
<th>SBR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading (pphr)</td>
<td>(\lambda)</td>
<td>(\sigma_T)</td>
</tr>
<tr>
<td>0_{o, 10, 20, 30, 40, 50}</td>
<td>6.85</td>
<td>117.9</td>
</tr>
<tr>
<td>20</td>
<td>5.80</td>
<td>124.6</td>
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<tr>
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<td>5.17</td>
<td>129.6</td>
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<tr>
<td>50</td>
<td>4.97</td>
<td>134.6</td>
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</tbody>
</table>

Figure 10.19: Biaxial Extension Ratio at Break.
Figure 10.20: Biaxial True Stress at Break.

If the combined results are examined it can be seen that there is very little difference between the biaxial extension ratios at break between the two polymers. It can also be seen that the extension at break reduces with carbon black loading. This latter observation is expected as carbon black will reduce chain extensibility by increasing the effective cross link density of the compound.

If the stress at break is examined it can be seen that at low carbon black loadings the strength of SBR is greater than NR. However whereas the strength of NR is almost constant with the addition of carbon black, SBR’s reduces. This reduction was not noticed however with decreased state of mix (increased effective carbon black loading). Rather the converse was found with increasing properties with increased effective carbon black loading. This can be seen in Figure 10.21.
These results indicate the reduction in strength of SBR could be linked to the finite extensibility of individual chains, like the extension ratio at break. This would reduce the biaxial strength as the stress carrying capacity of the compound will be reduced as the layers peel back when their chain extension limit is reached. This links with the state of mix results as increasing mixing increases the damage to the polymer chains, which in turn will reduce the stress at break due to the chains failing earlier.

Although there is little evidence of crystallinity occurring in NR during the biaxial test in Figure 10.20, there is an unexpected “S” shape in true stress at 300% extension, Figure 10.22.
This 'S' shape, attributed to crystallinity in the uniaxial test, could be due to the fibres seen to occur during delamination as these will be predominantly uniaxial. However X-ray reflection tests carried on NR Gum by Roberts et al\textsuperscript{12} indicated the presence of a different crystal structure in NR during a biaxial test than in uniaxial samples, although the nature of this structure was not investigated. It is perhaps the loss of these slight crystallisation effects with the addition of carbon black which accounts for the very slight increase in strength in Figure 10.20 as carbon black reinforcement replaces crystallinity.

No tests were carried out however by Roberts et al on SBR, and although the large change in SBR's properties is likely to be due to the greater extensibility possible in SBR without the hindrance of edge defects, the change may be due to a change in the structure of SBR under biaxial loading. Although Figure 10.22 shows no sign of crystallinity life NR with an almost linear rise in true stress at 300% extension, X-ray reflection tests would clarify the structure of both polymers during biaxial loading.

10.6 Summary and Comparison of Results.

In summary, NR when tensile tested has much greater stress and extension at break than SBR, although the values converge with carbon black loading. The effect of crystallinity can also be seen in NR. When tested biaxially SBR is much stronger (stress and elongation
at break) than NR at low carbon black loadings, again with the values converging with increased carbon black loading. NR does however show some evidence of continuing crystalline behaviour, with a slight s-shape when true stress at 300% extension is plotted. These trends, along with comparisons between the two tests, can be seen in Figure 10.23 and Figure 10.24.

Figure 10.23: Comparison of Extension Ratio at Break.

Figure 10.24: Comparison of True Stress at Break.
Comparing the uniaxial and biaxial extensions at break it can be seen that at less than 20pphr, the biaxial results are higher and very similar beyond 20pphr for all bar uniaxial SBR.

If we now examine the stress at failure, the loss of crystallinity biaxially in NR is clear, with a very slight increase in stress with carbon black loading compared to the uniaxial curve. SBR however shows a decrease with carbon black loading biaxially from an initially higher point.

These results indicate a possible change in failure mechanism between the uniaxial and biaxial tests. Uniaxially crack tip growth criteria appear to dominate. This is indicated by the increase in strength with carbon black loading and crystallinity as these both hinder crack growth (see Chapter 3.5). It also explains the very low strength of SBR at low carbon black loadings as this is prone to edge defects.

Biaxially the results, along with the observations on fracture patterns and surfaces, indicate the dominant failure mechanism is the extension limit of individual chains. As it appears that the sample the rubber may have differing levels of cure or a stress gradient through the thickness, failure of individual chains causes delamination. This failure layer by layer, with each layer peeling back, reduces the crack growth rate through the thickness of the sample. The formation of fibres (Figure 10.14) complicates matters as these are fundamentally uniaxial. However the presence of these fibres may explain the slight increase in strength of NR with carbon black loading in Figure 10.22 as these will fundamentally be under uniaxial loading.
References

1 BS903 Part A2, (1995)


4 C F Flint and W J S Naunton, *Trans. IRI*, 12, 367 (1937)

5 L R G Treloar, *Trans. IRI*, 19, 201 (1944)


10 P S Oubridge, *Private Communication*.


Chapter 11
Fatigue Test Results.

11.1 Introduction.

In this Chapter the fatigue test results are reported, although Chapter 10 should be read in conjunction for clarity. The tests were conducted after the improvements to the apparatus described in Chapters 8 and 9 were completed. Owing to the time taken to complete a series of fatigue tests, in this Chapter the results are in two parts only. The main results are on the same series of compounds as the strength test results in Chapter 10, along with the state of mix investigation compound.

In order to calculate a mean fatigue life for each test, a Weibull distribution has been used as described in Chapter 3.6 b). Such an analysis also provides an indication of the spread of lives by means of 10 % and 90% quantiles. These give an indication of the fatigue life at which 10% and 90% of samples respectively will have failed. However for statistical comparisons between the sets of data a Normal distribution has been used.

For clarity this Chapter is split into four main sections: 11.2 Tables of Results; 11.3 Analysis of Results; 10.4 Failure Modes; 10.5 Discussion of Results.
### 11.2 Tables of Results.

<table>
<thead>
<tr>
<th></th>
<th>N23 10%</th>
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<th>N33 50%</th>
<th>N33 90%</th>
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Table 11.1: Uniaxial Cycles to Failure (NR).
### Table 11.2: Uniaxial Cycles to Failure (SBR).

† Fatigue life at this strain level based on two failed samples out of eight.

<table>
<thead>
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<th>λ</th>
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### Table 11.3: Biaxial Cycles to Failure (NR).

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<th>N33 10%</th>
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### Table 11.4: Biaxial Cycles to Failure (SBR).

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### Table 11.5: Uniaxial Cycles to Failure for State of Mix Compounds.

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<th>moderate mix 50%</th>
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Table 11.6: Biaxial Cycles to Failure for State of Mix Compounds.

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<th>90%</th>
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Table 11.7: Effect of Cure on Uniaxial Cycles to Failure.

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11.3 Analysis of Results.

In order to assess the statistical viability of the results, the results from the experimental series were investigated further. Initially, the data were considered in turn as Normal and Weibull distributions and the resulting 10%, 50% and 90% quantiles compared in Table 11.8.
As may be seen from Table 11.8, the quantiles predicted by the two distributions are in general agreement, thus allowing the use of Normal statistics for further analysis of the fatigue data.

Given the inevitable spread of fatigue lives, all available data was used in the subsequent analyses. First the differences in fatigue life between extension ratios was examined. Using comparison of groups, the significance of differences for a given compound were assessed. If for any load case and compound the difference was not significant, no further analysis would be carried out. All the results however were found to be significant, and mostly highly significant, with the results summarised in Table 11.9. The groups correspond to extension ratio, and the degrees of freedom are those between and within groups.

<table>
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<tr>
<th>Compound</th>
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<th>Weibull Distribution</th>
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<td></td>
<td></td>
<td></td>
<td>10%</td>
<td>50%</td>
</tr>
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Table 11.8: Fatigue Life, Normal and Weibull Quantiles (x10^3).
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<td>Uniaxial</td>
<td>3 and 27</td>
<td>51.42</td>
<td>&gt;99.9%</td>
</tr>
<tr>
<td></td>
<td>Biaxial</td>
<td>2 and 17</td>
<td>24.42</td>
<td>&gt;99.9%</td>
</tr>
<tr>
<td>N33</td>
<td>Uniaxial</td>
<td>3 and 26</td>
<td>20.97</td>
<td>&gt;99.9%</td>
</tr>
<tr>
<td></td>
<td>Biaxial</td>
<td>2 and 18</td>
<td>4.42</td>
<td>97.3%</td>
</tr>
<tr>
<td>N53</td>
<td>Uniaxial</td>
<td>3 and 28</td>
<td>76.53</td>
<td>&gt;99.9%</td>
</tr>
<tr>
<td></td>
<td>Biaxial</td>
<td>2 and 11</td>
<td>189.84</td>
<td>&gt;99.9%</td>
</tr>
<tr>
<td>S23</td>
<td>Uniaxial</td>
<td>4 and 34</td>
<td>2.95</td>
<td>96.6%</td>
</tr>
<tr>
<td></td>
<td>Biaxial</td>
<td>2 and 15</td>
<td>7.65</td>
<td>99.5%</td>
</tr>
<tr>
<td>S33</td>
<td>Uniaxial</td>
<td>3 and 28</td>
<td>13.14</td>
<td>&gt;99.9%</td>
</tr>
<tr>
<td></td>
<td>Biaxial</td>
<td>1 and 12</td>
<td>90.51</td>
<td>&gt;99.9%</td>
</tr>
<tr>
<td>S53</td>
<td>Uniaxial</td>
<td>4 and 28</td>
<td>26.62</td>
<td>&gt;99.9%</td>
</tr>
<tr>
<td></td>
<td>Biaxial</td>
<td>3 and 19</td>
<td>31.05</td>
<td>&gt;99.9%</td>
</tr>
</tbody>
</table>

Table 11.9: Significance of Extension Ratio.

It was also found that when limited numbers of groups for a specific compound were compared, the differences are still significant apart from the uniaxial lives of S33 at extension ratios: 2.50 and 2.85.

Figure 11.1 and Figure 11.2 represent these results graphically. In these plots a solid line is used for the uniaxial test, and a dotted line for the biaxial test. The thicker lines are associated with the filled in symbols. The strength data are included as they represent single cycle fatigue.
Figure 11.1: Cycles to Failure for NR

Figure 11.2: Cycles to Failure for SBR.
Although not subject to the same statistical analysis, the effect of State of Mix on cycles to failure is shown in Figure 11.3.

![Figure 11.3: Effect of State of Mix on Fatigue Life.](image)

It can be seen in these figures that the fatigue life \( N \) of the compounds decreased exponentially with increasing extension ratio \( \lambda \), even to the one cycle strength test result, consequently:

\[
\log(N) = a + b \lambda
\]  

which is in the form of a single, linear regression equation.

The next analyses were undertaken to confirm this finding and compare, for each compound, its uniaxial and equibiaxial mean lives. In particular:

- is equation 11.1 valid for both uniaxial and equibiaxial modes?
- if so, do the data from both tests group around a single line?
- or should they be described by individual lines of different slope and intercept?

The method used was to construct an analysis of variance table for the two separate individual, uniaxial and equibiaxial, regression lines (ignoring the strength test result), compared to a single line. An example is shown as Table 11.10, for N23.
The “F” ratio is obtained by dividing the mean square due to the difference between the single and individual lines by the residual, or error, mean square. With 2 and 3 degrees of freedom, its value of 39.7 corresponds to a significance of 99.3%.

The summarised results of these analyses are given in Table 11.11.

Table 11.11: Comparison of Uniaxial and Equibiaxial Fatigue Data.
Apart from S23 under equibiaxial conditions, all the regression lines are very or highly significant. Comparing the uniaxial and equibiaxial lives, individual lines represent the behaviour of N23, N53 and S23. For N35, S33 and S53, the two sets of data fall around single lines, as indicated by the bracketed low difference percentage significances. These fits are illustrated in Figure 11.4 to Figure 11.9.

Figure 11.4: Cycles to Failure (N23).

Figure 11.5: Cycles to Failure (N33).
Figure 11.6: Cycles to Failure (N53).

Figure 11.7: Cycles to Failure (S23).
Figure 11.8: Cycles to Failure (S33).

Figure 11.9: Cycles to Failure (S53).
It may also be seen from these Figures that, where there is a significant difference between the lives under the two modes of deformation, the regression lines converge at low extension ratios, at about $\lambda = 2$, but diverge at the higher ratios. From the limited evidence at extension ratios above 2.0, the fatigue life of natural rubber compounds is greater uniaxially than equibiaxially; whilst the opposite is true for SBR.

To assess the differences between compounds, the regression equation of each was compared to those of the other compounds in turn. The percentage significances are listed in Table 11.12 for the uniaxial mode of deformation and Table 11.13 for the equibiaxial mode. The bracketed percentage significances, less than 95%, indicate a single log(life) versus extension ratio line best represents the two compound in question.

<table>
<thead>
<tr>
<th></th>
<th>S53</th>
<th>S33</th>
<th>S23</th>
<th>N53</th>
<th>N33</th>
</tr>
</thead>
<tbody>
<tr>
<td>N23</td>
<td>96.3%</td>
<td>98.1%</td>
<td>&gt;99.9%</td>
<td>96.6%</td>
<td>[87.6%]</td>
</tr>
<tr>
<td>N33</td>
<td>96.8%</td>
<td>[3.9%]</td>
<td>99.1%</td>
<td>95.0%</td>
<td></td>
</tr>
<tr>
<td>N53</td>
<td>[36.1%]</td>
<td>99.4%</td>
<td>&gt;99.9%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S23</td>
<td>&gt;99.9%</td>
<td>99.8%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S33</td>
<td>99.0%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 11.12: Comparison of Uniaxial Fatigue Lives.

As may be seen, only 3 out of 15 comparisons fail to differentiate between the two compounds involved.

<table>
<thead>
<tr>
<th></th>
<th>S53</th>
<th>S33</th>
<th>S23</th>
<th>N53</th>
<th>N33</th>
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</thead>
<tbody>
<tr>
<td>N23</td>
<td>97.4%</td>
<td>94.3%</td>
<td>96.9%</td>
<td>96.8%</td>
<td>99.1%</td>
</tr>
<tr>
<td>N33</td>
<td>98.3%</td>
<td>[86.6%]</td>
<td>[81.7%]</td>
<td>98.4%</td>
<td></td>
</tr>
<tr>
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<td>95.2%</td>
<td>97.7%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S23</td>
<td>98.6%</td>
<td>[88.6%]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S33</td>
<td>97.2%</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Table 11.13: Comparison of Equibiaxial Fatigue Lives.
For the equibiaxial mode of failure, 4 out of 12 comparisons fail to differentiate between the two compounds involved, with one marginal significance: [N23, S33].

Table 11.11 to Table 11.13 are summarised in Table 11.14 to Table 11.16. In these tables;

- I indicates the best fits are obtained with individual lines;
- C indicates a common slope;
- S indicates a single line fits the combined data.

<table>
<thead>
<tr>
<th></th>
<th>N23</th>
<th>N33</th>
<th>N53</th>
<th>S23</th>
<th>S33</th>
<th>S53</th>
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</thead>
<tbody>
<tr>
<td>N23</td>
<td>I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>N33</td>
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<td></td>
<td>I</td>
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</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td>I</td>
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</tr>
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<td></td>
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</tr>
<tr>
<td>S53</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>S</td>
</tr>
</tbody>
</table>

Table 11.14: Uniaxial versus Equibiaxial Fatigue Lives.

<table>
<thead>
<tr>
<th></th>
<th>S53</th>
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<tbody>
<tr>
<td>N23</td>
<td>S</td>
<td>C</td>
<td>I</td>
<td>I</td>
<td>S</td>
</tr>
<tr>
<td>N33</td>
<td>C</td>
<td>S</td>
<td>I</td>
<td>C</td>
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<td>C</td>
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<td>I</td>
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<td></td>
</tr>
<tr>
<td>S33</td>
<td>C</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Table 11.15: Uniaxial Fatigue Lives.
11.4 Failure Patterns.

a) Crack Initiation.

During a tensile test, crack initiation usually occurred at an edge defect and propagated through the material. This has also been observed during uniaxial fatigue tests, with uniaxial samples exhibiting large cracks before failure. The cause of crack initiation in biaxial samples appeared to be an external surface defect, although an internal defect cannot also be ruled out as a cause. These cracks do not necessarily start at the pole however. At large extensions (energy levels), biaxial failure once the crack had propagated through the sample was usually catastrophic, a single split forming from the flaw radially outwards, often over the pole. At smaller extensions however failure usually occurred due to pressure loss through the hole formed by the crack.

However like the strength tests (Figure 10.13), at low strains a large degree of delamination was also apparent prior to failure on some biaxial samples. This occurred either on the outside or inside surface, although it would appear that cracks only propagated from the outside flaws. Such a failed sample is illustrated in Figure 11.10;

<table>
<thead>
<tr>
<th></th>
<th>S53</th>
<th>S33</th>
<th>S23</th>
<th>N53</th>
<th>N33</th>
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<td>I</td>
<td></td>
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<tr>
<td>N53</td>
<td>C</td>
<td>I</td>
<td>I</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S23</td>
<td>C</td>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S33</td>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 11.16: Equibiaxial Fatigue Lives.
b) **Fracture Surfaces.**

When the surfaces of the failed fatigue samples were examined a difference between the surfaces of NR and SBR samples became apparent. Generally the fatigued surfaces of SBR samples were visibly coarser than NR, with the uniaxial samples usually having a large smooth triangular area at one corner. Such a surface can be seen in Figure 11.11.
A failed NR dumbbell has a much smaller smooth area, usually along one edge. Also at larger extensions a ridged appearance is also apparent, as can be seen in Figure 11.12.

Figure 11.12: Failed NR Uniaxial Fatigue Sample (N23, 45% of Elongation at Break).
Such ridges may be analogous to the ridges seen on the edges of biaxial strength test petals (Figure 10.15), and may be an indication of knotty tear occurring. However when a large number of samples are compared, unlike the biaxial strength test petals, the ridges appear to be more random. A more likely explanation for the ridged pattern seen on fatigued samples is the molecular bundling at crack tips observed by Goldburg\textsuperscript{1}.

The difference between the two fracture surfaces is also apparent in biaxial fatigue samples. Looking at NR, semi-circular ridges can be seen (Figure 11.13) through the thickness of the sample, probably caused as the crack propagated

![Image of biaxial flaw in NR (N53)](image)

Figure 11.13: Biaxial Flaw in NR (N53).

With SBR however the same semi-circular crack can be seen (Figure 11.14), but without the ridges.
The smoother pattern in NR is probably due to the more controlled failure mechanism that occurs during stick slip. SBR, as described in Chapter 3.3, undergoes time dependant failure where the of crack continues to grow even when the applied extension remains constant. This difference in crack growth mechanism explains the larger final fracture area with the uniaxial SBR samples. As the crack progresses, the energy density increases, hence the crack can grow more each cycle as the test progresses. With NR though, although the energy density also increases, so does the degree of crystallisation. This restricts the increase in crack growth, hence the smaller final area.

The semi circular biaxial flaws are also interesting. These flaws, and hence failure, do not only occur at the pole, resulting in general rather than equibiaxial conditions during crack growth. This alternate starting point also indicates that the cracks may not be initiated by a point of maximum stress, rather a localised weakness or stress raiser, unlike the biaxial strength case. The general biaxial loading of the sample in the area of the crack, along with the nature of the test, means the cracks remain oval under stress, with cracks growing both in circumference, and also depth. This semicircular crack growth can be seen in Figure 11.13 and Figure 11.14.

Failure occurs when this crack propagates through the sample, with the type of failure depending on the extension ratio under test. At high strains, due to the higher energies
present in the samples, failure occurs as explosive decompression, with enough tearing 
energy to propagate the tear up the stress gradient and across the pole. At lesser extensions 
(energies) failure occurs due to pressure leaks, and the test may be continued if the pressure 
is raised (not done for these tests). In both cases however, owing to the multi directional 
crack growth, there is less tearing energy on each direction, increasing fatigue life.

11.5 Discussion of Results.

a) Equipment Limitations.

Of the apparatus, the uniaxial machine was the more difficult to set up, but the scope for 
error was less. Although after 100 cycles, the specimens were adjusted to remove slack due 
to permanent set, in many cases further permanent set occurred once the machine was 
started again. This was usually ignored, unless the additional extension was causing the 
samples to “bow” excessively at bottom dead centre (BDC).

Although easier to initially set up, the accuracy of the biaxial fatigue machine is difficult to 
quantify. Owing to the nature of the method chosen, the test was closely linked to the 
biaxial strength test to give extensions for a given height. This closeness meant that 
allowance for permanent set was not a simple matter as data at low heights was not 
available. Because of this no allowance was taken, effectively making the calculated 
extension too large.

The biaxial test also had only crude rate control. Although limited tests on uniaxial samples 
indicated little effect, the biaxial test results suggest a larger rate effect. One example is the 
change in the failure patterns of samples cast from NR latex with increased test speed 
described in Chapter 10.5 c). It is also probable that due to this crude rate control, the 
biaxial and uniaxial tests were carried out at different test rates.

For the purposes of this work however, the possible largest error was the load condition at 
failure. As indicted in section 11.4 b), final failure did not often occur at the pole. This 
means the load case in the area of failure was multiaxial (general biaxial), rather than the 
equibiaxial required. Although these results do show distinct changes in fatigue life under 
multiaxial loading, they may not allow true comparisons between uniaxial and equibiaxial
fatigue lives. However in most cases, the localised loading would have been very close to the required equibiaxial loading as failure usually occurred close to the pole.

b) Discussion of Results.

If we look at the results, the most striking feature is the apparent linear log relationship between fatigue life and applied extension highlighted in the statistical analysis in Section 11.3. If this holds at smaller extensions, it would allow easy prediction of fatigue life from a few samples.

It can also be seen in Figures 11.1 to 11.3 that for a given extension fatigue life is increased as carbon black loading increases. Likewise improved state of mix, better black dispersion, also increases fatigue life, despite the reduced effective black loading. The increase in fatigue life with carbon black loading may be ascribed to the carbon black reducing crack growth\(^2\). This reduction in crack growth is attributed to frictional energy losses as the rubber molecules stretch over black particles, and the attachment of 'loose' chain ends to carbon black particles. This latter effect reduces crack growth by reducing the crack surface area, and making the broken chain load bearing.

The increased fatigue life with increased dispersion is probably due to the smaller aggregate particles reducing the stress magnification effect of carbon black particles\(^2\). Both this and the crack slowing effects described above are more important in SBR due to the time dependant nature of crack growth in SBR\(^3\). This can be seen in the larger increase in fatigue life in SBR with black loading (Figure 11.2) compared to NR (Figure 11.1).

The biaxial fatigue life of NR is reduced compared to the uniaxial load case, probably due partly to the loss of crystallinity. However this contradicts the expected longer fatigue life from the circular cracks. SBR however sees a marked increase in fatigue life biaxially for a given extension ratio in both the state of mix and experimental mixes. This increase in the fatigue life of SBR may be due to the lack of edge defects. However as the extension at break is larger in SBR in the biaxial test, Table 10.18, compared to its uniaxial value, Table 10.9, a given extension is a lower percentage of the elongation at break than for NR. The crack growth mechanism observed also helps SBR as the cracks are blunted during each cycle, reducing the local stress.
These results indicate that to increase fatigue life:

- NR requires crystallisation to occur.
- SBR requires an absence of edge defects.

Also the fatigue life of both polymers increases with increased carbon black loading.

If the energy function proposed by Roberts et al. is considered, an indication of the crack growth rate is possible. This expresses the fatigue life as a function of the cut growth constant, $B$, the strain energy density, $W$, and extension ratio, $\lambda$. This is described by Equations 11.2 and 11.3 for the uniaxial and equibiaxial load cases respectively (see Chapter 3.4b).

\[
N = \frac{1}{(\beta - 1)Bc_0^{\beta - 1}}(2kW)^{-\beta} \quad -11.2
\]

\[
N = \frac{1}{(2\beta - 1)B'c_0^{(2\beta - 1)}}(2k^2W)^{-\beta} \quad -11.3
\]

Thus if log life is plotted against log $2kW$ (or $2k^2W$), the energy function, as suggested by Roberts et al. $\beta$ (the gradient) may be calculated. However, calculation of $B$ (inversely proportional to the intercept) is not easily possible as the initial flaw size needs to be known. However both a shallow gradient and a high intercept indicate low crack growth rate.

Such energy plots are shown in Figure 11.15 and Figure 11.16;
It can be seen in these Figures that there are two slopes, as expected from the work of Roberts et al\textsuperscript{4}. This indicates that the crack growth rate changes radically at a given extension, indicating a probable change in failure mechanism. This could possibly indicate
a change to knotty tear, although there is no evidence of a change in the nature of the fracture surface when they were examined. It can also be seen that the energy level, as expected, changes with test and compound.

For both polymers the fatigue life is higher with biaxial loading for a given energy function. This contrasts the reduction in fatigue life in NR for a given extension ratio, Figure 11.1. This change could be due to the much larger strain energies present in the biaxial strength test, Table 10.14, compared to a uniaxial test, Table 10.3. It is also apparent in the larger shift to the right in the energy plots between the uniaxial and biaxial result compared to Figure 11.1 and Figure 11.2.

The energy function plots do however show the expected reduction in crack growth rate with biaxial loading. Although there is a slight increase in gradient for the biaxial result, there is a much larger increase in the intercept value. The intercept, although indicative of reduced crack growth rate, may also indicate a reduction in initial flaw size as this is also inversely proportional to the intercept. Either indicates improved biaxial fatigue performance compared to the uniaxial load case, although this may only be at equivalent strain energy levels.
References

1 A Goldberg and D R Lesuer, Rubber Chem. and Tech., 62, p288 (1989)


Chapter 12
Finite Element Analyses.

12.1 Introduction.

From the outset of this research, one of the aims was to relate the results to finite element analysis (FEA). It was hoped that the results gained would allow more precise interpretation of the results from an FE analysis in terms of product performance. In the process of the project however, the development of an alternative elastic theory became important, and this chapter describes its application into a commercial FE code.

The code chosen to investigate this application of the Filament theory, and for other FE work required was NISA II from EMRC¹. This was selected over the better known MARC² and ABAQUS³ software as it was available to run on a PC at the time. It also had the advantage that members involved in the project had used the software in the past. Capable of hyperelastic analyses, with good pre and post processors, it was hoped that the package would provide a cost effective route into FEA.

In this chapter the initial work to investigate NISA’s facilities is described, followed by application of Filament constants to simple models, then to more representative models.

12.2 Evaluation of Software.

Before any detailed practical work could be undertaken, the capabilities of NISA needed to be evaluated. This was undertaken while the equipment was being manufactured, and it was hoped would provide guidance for later work.

For this initial investigation a model of the BS type 2⁴ dumbbell was created, shown in Figure 12.1;
As can be seen, in order to speed analysis times only a $\frac{1}{4}$ model was produced, with boundary conditions applied to simulate the rest of the sample. The loading applied was also representative with the model fixed at the tab, and force or displacement applied to the gauge. Also, as no real elastic constants were available, a simple Mooney Rivlin model was used (Equation 2.22), with a $C_{10}$ of 0.6 and a $C_{01}$ of 0.1.

When the results from the trials were compared, it was found that on this model that the different options had little effect of the results. The only difference was found to be solution time, with substantial differences with some of the revised iterative schemes. However one result was thought to be useful. With this model, regardless of the pairs of elastic constants used, 73% of the overall extension occurred in the gauge length. This, it was initially thought, would be useful to simplify the fatigue tests as described in Chapter 5.3. Unfortunately this was not found to be the case experimentally, probably to the limited range of constants used.
Although these results were of limited use with later analyses, they were useful in gaining an understanding of the solution options available, and the use of the software.

### 12.3 Application of Filament Theory.

**a) Initial Trials.**

In order to evaluate the accuracy of the constants calculated from the Filament Theory, a simple FE model was generated. This model, Figure 12.2, was a simple rectangle subjected to both uniaxial and equibiaxial loading to 350% extension, with elastic constants derived from data for NR and SBR with 20 and 50pphr respectively of N330. For these materials two sets of elastic constants were used, one set derived from uniaxial data alone, and another set derived from multiaxial data (uniaxial and equibiaxial). In both cases a 3rd power invariant function was used (see Table 2.1) as this gave a good degree of fit over the whole extension range. The constants used are listed in Table 12.1.

![Figure 12.2: Simple Uniaxial FE model.](image)
It was found that the results from the uniaxial models closely matched the uniaxial experimental results for both constant sets. This can be seen in Figure 12.3.

However when the results from the biaxial models were compared with the biaxial experimental result, differences were found at extensions beyond 100% as can be seen in Figure 12.4;

### Table 12.1: Rivlin Elastic Constants Used for FEA

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<th>Compound/Type</th>
<th>$C_{10}$</th>
<th>$C_{01}$</th>
<th>$C_{20}$</th>
<th>$C_{02}$</th>
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<td>-1.13E-4</td>
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<td>3.21E-02</td>
<td>-7.54E-06</td>
<td>2.31E-04</td>
<td>1.77E-08</td>
</tr>
</tbody>
</table>

Figure 12.3: Comparison of FE Models and Experimental Results

(Uniaxial Load Case, S53)
In the figure it can be seen that the model using only uniaxial constants diverges increasingly from the experimental data with increased extension. Conversely the model using elastic constants derived from multiaxial data remains close to the experimental result, even at 350%. Although of interest as it compares the two sets of constants, it is of limited use. Because of the test method chosen, a finite difference approach was needed to calculate the stress strain data from the measurements taken in the biaxial test. As the finite difference technique is also employed in FE analyses, we are therefore only comparing in effect one simulation with another. A more interesting result would be simulating the real tests.

b) Representative FE Models.

In order to more fully test the accuracy of the derived elastic constants, it was decided to apply them to representative models of the tested geometry's. The elastic constants used for Section a) were reapplied on three models: the ¼ Dumbbell model used for the initial evaluation, Figure 12.1; a simple axi-symmetric model, Figure 12.5; a more complex three dimensional ¼ equibiaxial model, Figure 12.6.
To simulate the loading conditions both equibiaxial models were fixed at the outer edge, but allowed to pivot, with appropriate edge conditions to simulate the rest of the model. The pressure was applied in the form of follower pressures which stay perpendicular to the surface, which was linearly increased to the desired value.

Unfortunately none of these models worked using the material constants in Table 12.1 much beyond 100 to 150%, despite support from the supplying company. The problems were mainly concerned with convergence on one or more criteria, and element distortion with
the biaxial models. Unfortunately it was decided, both by ourselves and the supplier, that NISA II is not really suitable for rubber models at high strains.

However if the elastic model was reverted back to the two term Mooney Rivlin function, the axysymetric model was found to work to the extensions required, as can be seen in Figure 12.7.

Figure 12.7: Inflated Equibiaxial FE Model

Unfortunately, owing to the late stage in the project that truly representative elastic constants were available, it was not possible to try to solve such problems on other FE packages.
References

1 Wilde and Partners, Stockport, Cheshire

2 MARC UK Ltd., 35 Shenley Pavillions, Chalkdell Drive, Shenly Wood, Milton Keynes, MK5 6LB

3 HKS UK Ltd., The Genesis Centre, Birchwood, Warrington, WA3 7BH

Chapter 13
Conclusions and Recommendations.

13.1 Conclusions.

Significant differences were found between the uniaxial and equibiaxial (biaxial) behaviour of rubber compounds, especially SBR. In uniaxial extension the effect of crystallinity is apparent in NR with an ‘S’ shaped increase in true stress at 300% extension with black loading, caused as the black and crystallinity interact. SBR though shows a marked increase in strength (stress at break) and stiffness (true stress at 300% extension) with black loading. However when the effect of test rate and black type were investigated, no statistical difference was found between the test results, although the lack of difference with black type may be due to the degree of scatter in the results.

In biaxial extension though a difference between black types was apparent, along with a possible, unquantified, rate effect. Also, under biaxial loading, the results for both NR and SBR were closer, with SBR stronger than NR at low black loadings. However there is some evidence that even in a biaxial test crystallinity may still have a slight effect on NR. This is evident in an ‘S’ shaped curve for biaxial true stress at 300% extension similar to that found with the uniaxial test, although it is less pronounced.

When the uniaxial and biaxial strength tests are compared, the extensions at break for all bar uniaxial SBR are close, especially above 20pphr. There is also a marked increase in the biaxial strength of SBR, although the trend with black loading was found to be reversed. In NR however, due to the marked interaction of black and crystallinity evident in the uniaxial results, there is no apparent trend between the uniaxial and biaxial results with black loading.

Although tests were limited, distinct differences were also found in fatigue between the two load cases. In all cases there was the expected increase in life with increased black loading, attributed to reduced crack growth rate. This increase in life was enhanced in the biaxial test (for a given energy function), which along with the introduction of a difference between black types in strength tests indicates the biaxial test may be more susceptible to
particulate reinforcement. It was also noticed that a simple log-linear relationship may
relate life and extension ratio.

The effect of load case however depends on the chosen $\chi$-axis. When plotted against
extension ratio the life of SBR was found to increase with biaxial testing, while the
converse is true for NR. This decrease the biaxial fatigue life of NR is probably due to the
loss of crystallinity, whereas the increase in the life of SBR may be due to the lower
percentage of the failure extension at a given extension ratio in a biaxial test. Also, owing
to the time dependant nature of crack growth in SBR, the increase in life may be due to the
reduction in stress concentration at the crack tip due to the more uniform load distribution.

However if life is plotted against a function of strain energy, the life of both polymers
increases in the biaxial test for a given energy. This may be due to the increased strain
energy present in a biaxial sample for a given extension, although the graphs indicate a
reduction in crack growth rate in the biaxial load case.

From the results to increase life;

• NR requires crystallisation to occur.
• SBR requires no edge defects.

Also life in both polymers increases with black loading.

This increase in the biaxial properties of NR and SBR is very interesting considering the
increased strain energy and effective elongation of rubber molecules during a biaxial test.
These results, along with observations during testing, suggest the cause of crack initiation
and growth differs between the uniaxial and biaxial tests both for strength and fatigue. In
Uniaxial strength and fatigue tests, the crack grows across the sample from an initial edge
defect. This, along with the mode of crack growth, is as expected from the literature. The
mode of failure in an inflated diaphragm however appears to depend on the test type. With
fatigue, failure occurs initially from a surface defect from which a semicircular crack then
propagates through the sample. Once this stage is reached either catastrophic failure or
pressure loss occurs depending on the strain energy in the sample.

In the strength tests it appears that failure is preceded by delamination, with the crack
growth rate through the sample governed by the extension limits of individual chains.
Chapter 13

Catastrophic failure then causes an individual pattern for each compound which falls into two main groups, radial and cloverleaf. These both consist of 'petals' which form from the pole to the base, the former thin with straight edges, the latter fatter with curved edges. The size of these 'petals' is governed partly by black loading and possibly test rate. At this time however it is unclear whether these patterns are caused by the known mechanisms of stick slip and knotty tear etc., or a different mechanism unique to this type of biaxial test. However the possible presence of crystallinity in NR, plus the possibility of an unknown change in the microstructure of SBR, may also play a part.

Although the third aim in Chapter 1, Application to FEA, was not entirely met, the unsuitability of elastic constants base don uniaxial data alone for multiaxial finite element analyses was highlighted. Instead a novel elastic theory was utilised which represents the behaviour of rubber using a single elastic filament. This, the Filament Theory, could not however completely simulate the inflated diaphragm. It needed an experimentally derived scaling factor to match the simulated and measured pressures. It did however accurately predict the profile of the inflated diaphragm and the extension at the pole using uniaxial data alone. Because of this the theory became a useful tool for the project and, with the additional experimental factor, was applied successfully to simple, large strain, FE models.

13.2 Recommendations.

a) Further Work.

This work has given a good indication of the substantial differences between the uniaxial and biaxial performance of rubber compounds. However further testing in certain areas would confirm the findings, and answer some present questions.

- Both biaxial tests require further development in order to improve the reliability and overall confidence in the results obtained. Ideas for such development are given in Sections b) and c).

- Further work is required to investigate the mode of failure during a biaxial test. This could possibly be achieved using high speed photography to capture the moment of failure. It is felt that such an investigation would give a better understanding of the micro- and macroscopic changes in rubber under multiaxial loading.
• X-ray reflection tests during biaxial testing would clarify the structure of rubber under biaxial loading. Of particular interest is the possibility of an organised, crystal like, structure in SBR.

• Further development of the Filament theory may also increase its accuracy, and confidence in its use. At present the variation of $\beta$ with varying degrees of multiaxial loading is unknown, along with it’s connection with strain invariants. A mathematical study would check the latter, and general biaxial testing would provide the mathematical form of the former

b) Development of Biaxial Strength Tester.

Although the concept of the apparatus and associated theory has been proven, further work is required in the following areas in order to develop the ease of use and accuracy of the equipment;

• Equipment revisions
• Software integration
• Electronic enhancement.

Dealing with each in turn;

i) Equipment Revisions.

Some revisions to the equipment would make it more commercially acceptable, although they will require a substantial redesign. First is conversion to a hydraulics rather than pneumatics. This would solve several problems with the current apparatus, and give some additional features.

• Reduced noise levels
• Improved rate control.
• The addition of volume measurement which should remove the need for an expensive laser distance device and also simplify the electronics.
• The ability to measure compound performance in harmful fluids or at elevated temperatures, for example gasket or hose material for automotive use.
• Increased ease of use.

Secondly alterations to the base may be worthwhile due to problems with the current clamping mechanism. At present the clamping mechanism can suffer from slippage, and is cumbersome to use. This could be helped by active hydraulic clamping, which would serve two purposes, keeping a uniform clamping force regardless of deformation, and also ease of use.

ii) Software Modifications

Presently analysis of the biaxial test results is a four stage process;

• Fit elastic constants to uniaxial data
• Get biaxial test data
• Analyse data to give average heights and pressures
• Simulate test to calculate stresses and extensions.

This is time consuming, and cancels some of the advantages of the current test. However, with an increase in computing power, the computer used for this research was a 386 based machine, these steps can be combined into a single package requiring only uniaxial data as input. Such software would function as follows;

1. Uniaxial data, either in the form of elastic constants or raw data is supplied, along with the specimen thickness.
2. The apparatus then conducts the test, measuring pressure and either volume or height.
3. The test readings and elastic constants are then used to calculate stress and extension.
4. Finally elastic constants may be derived from the uniaxial and biaxial data combined.

The above, combined with the hardware alterations, will increase the ease of use of the equipment to that approaching a conventional tensile tester.

iii) Electronic Enhancement.

At present the electronics of the system are not ideal, although they are basically sound. They are however prone to noise due the nature of the environment, especially in the final cable stage from the control box to the analogue to digital converter (A to D). It would be
advantageous to shorten and simplify this data path to reduce the possibility of noise. This
could be done by introducing an external A to D, with a serial link to the computer, as used
by some tensile testers. This offers other advantages than just a reduction in signal noise, as
it makes the set up more “plug and play”, requiring only the software, any (fast enough)
computer, and the external apparatus.

c) Development of Biaxial Fatigue Tester

Of the two pieces of apparatus, the biaxial fatigue tester is closer to final design. At present
it is completely pneumatic, requires large flow rates of air, and has a short fatigue life itself
(around 3 years). Ideally the following would make a simpler, longer lasting machine;

• Electronic logic to control the oscillator and cycle counting
• Electronic rather than pneumatic counters.

As the machine is basically sound, and the logic designed, it is felt that these alterations
would be relatively simple, especially the latter.
Appendix A

Computer Programs Written for Project.

A.1 Introduction.

Most of the tests undertaken for this project have yielded large quantities of data, often in "system" units which need converting before use. Because of this, several computer programs have been written to both simplify the data and convert them to engineering units, often for use in other software. All the software has been written in Microsoft® QuickBasic, with its facilities to identify and correct coding and logic errors.

This appendix describes the software and, in some key cases, indicates the development path taken by the software. Where appropriate, the software listings are contained in subsequent appendices.

A.2 Uniaxial Test Analysis.

a) Description of Package.

This software, TNRHOUN.BAS, is used to analyse the output from the Hounsfield tensile tester. Initially this consisted of only six pairs of stresses and extensions, which was insufficient and lead to erroneous results in some cases. However, an update of the tester operating software allowed up to 1000 data points to be saved, dramatically improving accuracy. It is this later version of the software that will be described.

The program "TNRHOUN.BAS" first examines these data for the number of repeated samples and selects 25 evenly spaced pairs for each. These selected data are processed to give the coefficients of either the original or modified Turner function. It also notes the failure points and their average.

Before the data are processed, the 25 pairs of stress and strain are displayed as points on a graph to allow the user to ignore any sample obviously in error, before elastic constant fitting is attempted. To determine these, options are presented: to enter estimates; scan estimated ranges; and adjust the coefficients individually. The fitted curve is then displayed, and again the constants can be adjusted by eye before saving.
b) Flow Chart.

Figure A.1: Flow Chart for “TNRHOUN.BAS”
A.3 Biaxial Test Data Acquisition and Initial Analysis.

a) Description of Packages.

Both to speed up data acquisition and to simplify the data, software packages were required. “BXLAS.BAS” controls and reads pairs of height and pressure from the test apparatus at about 100ms intervals, saving them to disk. “BXANAL.BAS” is then used to simplify the data. This is achieved by correcting the pressures for sample thickness, and allowing for rejection of bad tests. 25 evenly spaced pairs of data are then picked and the data are then averaged by polynomial curve fitting to give an average curve and failure height and pressure for later use.

In this appendix only “BXLAS.BAS” will be described further.

b) Flow Chart for BXLAS.BAS.

![Flow Chart for BXLAS.BAS](image)

Figure A.2: Flow Chart for “BXLAS.BAS”
A.4 Diaphragm Simulation.

a) Description of Package.

As the chosen biaxial test method (see section 4.2) does not measure stress and extension directly, a method of calculating them from the available measurements together with elastic constants, was required. The “SIMDIA.BAS” software does this using an iterative finite element technique based on that devised by James at al described. The results been checked for selected compounds using directly measured experimental data.

b) Flow Chart.

Figure A.3: Flow Chart for “SIMDIA.BAS”
c) Detailed Description.

i) Basic Equations.
An inflated profile is specified by selecting the equibiaxial extension ratios at the pole: \( \lambda_e \).
Because rubber is assumed incompressible, compared to its ability to distort, the extension ratio through the thickness is given by:

\[ \lambda_3 = \frac{1}{\lambda_1 \lambda_2} \]  

where \( \lambda_1 = \lambda_2 = \lambda_e \)

The Filament Theory (see Chapter 2.2 c) ii, given the three principal extension ratios, returns the biaxial true stresses \( \sigma_1 \) and \( \sigma_2 \). The tensions per unit width are then:

\[ T_1 = \lambda_3 \cdot t \cdot \sigma_1 \]  
\[ T_2 = \lambda_3 \cdot t \cdot \sigma_2 \]

where \( t \) is the original thickness.

From the theory of a doubly curved membrane\(^2\):

\[ P = K_1 \cdot T_1 + K_2 \cdot T_2 \]  

where \( K_1 \) and \( K_2 \) are the principal curvatures, and \( P \) is the internal pressure. Also, if a small radial segment of the inflated diaphragm is approximated as a segment of a spherical "cap", then:

\[ P = 2 \cdot K_2 \cdot T_1 \]

ii) Construction of a Diaphragm's Profile.
Let the uninflated radius of the diaphragm be \( R \) and its unstrained thickness \( t \).

1. For the first chosen \( \lambda_e = \lambda_1 = \lambda_2 \), assume the curvatures:
\[ K_4 = K_1 = K_2 = \frac{1}{R} \]  

although for the subsequent extension ratios, the previous pole curvatures are assumed.

2. Using the Filament Theory, the principal pole tensions: \( T_c = T_1 = T_2 \) are calculated. Then the internal pressure is:

\[ P = 2 \cdot K_c \cdot T_c \]  

The procedure for generating the profile of the diaphragm for the chosen extension ratio at the pole \( \lambda_c \) is then as follows.

3. At the pole:

\[ x = y = a = 0 \]  

where \( x \) and \( y \) are the co-ordinates, and \( a \) is the meridional angle, thus:

\[ \sin(a) = 0 \]  

and

\[ \cos(a) = 1 \]  

The profile is then generated as a succession of short arcs, as shown in Figure A.4.
Figure A.4: Construction of Diaphragm Profile

4. Divide the diaphragm into "n" annuli of equal width and consider each in turn, so that:

\[ da = \lambda_1 \cdot dR \cdot K_1 \quad \text{-A.9} \]

where \( da \) is the included angle of the arc representing the original annulus of width:

\[ dR = \frac{R}{n} \quad \text{-A.10} \]

\( a = a + da \)

the new meridional angle,

\[ dx = \frac{\sin(a) - \sin(a_p)}{K_1} \]

and:

\[ x = x + dx \]
the new x co-ordinate,

\[ dy = \frac{\cos(a) - \cos(a_p)}{K_1} \]

and:

\[ y = y + dy \]

the new y co-ordinate, where the subscript "p" indicates the previous meridional angle.

Also:

\[ \lambda_2 = \frac{x}{(i - 1) \, dR} \]

the new circumferential extension ratio, where \( i \) is the end point of the arc, starting at

\( i = 1 \) at the pole.

The new meridional extension ratio \( (\lambda_1) \) is found by iteration, as follows.

5. Using the Filament Theory, calculate \( \sigma_1 \) and \( \sigma_2 \) to give \( T_1 \) and \( T_2 \), then, by resolving

vertically over the portion of the diaphragm's profile constructed so far:

\[ P' = \frac{2 \cdot T_1 \cdot \sin(a)}{x} \]

where \( P' \) is a new estimate of the internal pressure. Recalculate \( \lambda_1 \) using:

\[ \lambda_1 = \lambda_{1, \, \left[ \frac{P}{P'} \right]^{0.3}} \]

and continue to iterate until \( P' \) is sufficiently close to the known pressure \( P \).
6. It follows that:

\[ K_2 = \frac{P}{2.T_1} \]

and:

\[ K_1 = K_2 \left[ 2 - \frac{T_2}{T_1} \right] \]

7. Finally:

\[ \sin(a_p) = \sin(a) \]

and:

\[ \cos(a_p) = \cos(a) \]

are prepared ready for the next annulus.

8. The steps 1. to 7. are based on the original estimate for \( K_1 \) which will almost certainly be in error, judged by whether the final \( x \) co-ordinate equals the radius of the uninflated diaphragm. To find the correct value for \( K_1 \), a three stage approach is used.

(i) A second value for \( K_1 \) is chosen so that the two final \( x \) co-ordinates straddle \( R \).

(ii) A third value for \( K_1 \) is found by linear interpolation, giving a third \( x \) co-ordinate.

(iii) Depending upon this third \( x \) co-ordinate, the upper and lower values for \( K_1 \) are modified to reduce the range, and so on until \( x \) is sufficiently close to \( R \).

d) Calculation of Equibiaxial Factor (\( \beta \)).

Once the initial run is completed with \( \beta=1 \), the \( \beta \) for a given extension can be calculated as follows if required:

(i) Estimate an initial value of \( \beta \) assuming the relationship;
(ii) Linearly interpolate between the pressures calculated in the previous two iterations to estimate a revised value of $\beta$.

(iii) If required repeat stage (ii) until the simulated and measured pressures match.

Please note that it is better to straddle the measured pressure with the first two iterations as this usually reduces the number of iterations required.

A.5 Elastic Constant Fitting and Conversion.

The elastic constants of a chosen expansion of the Rivlin strain energy function are determined using one of the following:

- an effective true Young’s modulus;
- the parameters of the original Turner function;
- the parameters of the modified Turner function.

The first option is limited to strains of about 100% because it is assumed a linear relationship exists between true stress and strain\(^3\). The latter two, however, can be used to high strains but require an equibiaxial factor, as described in Chapter 7.3 b).

The method is based upon creating a matrix of biaxial extension ratios and true stresses, followed by multiple regression to extract the elastic constants. More details are given in Chapter 9.5 a).
References


Appendix B
RUPEC.BAS

***************************************************************************
' RuPEC QBASIC Subroutine Library
'
' (c) J Hallett, P S Oubridge 1996
***************************************************************************

DECLARE SUB cenprt (ln%, btitle$)
DECLARE SUB choose (mn%, items%, key$, sel%)
DECLARE SUB delay (dt)
DECLARE SUB FileOpen (ext$, flnm$)
DECLARE SUB headframe (tr%, lc%, wd%, dp%)
DECLARE SUB lines (k%)
DECLARE SUB menu (btile$, tr%, lc%, sel%)
DECLARE SUB openscr (line1$, line2$, line3$, name$)

DECLARE FUNCTION affirm$ (ln%, col%, text$)
DECLARE FUNCTION atitle$ (ln%, col%, tl%)
DECLARE FUNCTION datemod$ ()
DECLARE FUNCTION datval! (vnum$, iok%)
DECLARE FUNCTION drive$ (ln%, col%)
DECLARE FUNCTION ok$ ()
DECLARE FUNCTION readkey$ ()

DIM SHARED mon(12) AS STRING * 3
DIM SHARED item$(10)

***************************************************************************
' Data Statements
***************************************************************************
'
months: DATA "Jan","Feb","Mar","Apr","May","Jun" DATA "Jul","Aug","Sep","Oct","Nov","Dec"

-----SUBROUTINES-----

FUNCTION affirm$ (ln%, col%, text$)
' Accepts a question and returns a "y/n" answer.

DO
  LOCATE ln%, col%
  n% = LEN(text$) + 11 'total length
  PRINT SPACE$(n%) 'clear space
  LOCATE ln%, col%
  PRINT text$ + "? y/n: "; 'question
  a$ = ok$ 'get reply
  LOOP UNTIL (a$ = "y" OR a$ = "n") 'only "y" or "n"
  affirm$ = a$ 'return answer
END FUNCTION

FUNCTION atitle$ (ln%, col%, tl%)
' Enters a title with a maximum number of characters, with:
' ln% = line; col% = column of first character entered;
' tl% = maximum length of the title.
' Note: > and < are displayed at either end of the input zone.

LOCATE ln%, col% - 1 'before input zone
PRINT ">";
LOCATE ln%, col% + tl% 'after input zone
PRINT "<";
LOCATE ln%, col%
PRINT SPACE$(tl%)
'clear input zone
ic% = 0 'zero character counter
title$ = "" 'set null string
DO
  ch$ = readkey$ 'enter characters
  IF (ASC(ch$) = 13) THEN 'RETURN> finished
    EXIT DO
  END IF
  ch$ = LTRIM$(ch$)
  title$ = title$ + ch$ 'concatenate characters
  ic% = ic% + 1
  IF ic% = tl% THEN 'maximum length reached
    EXIT
  END IF
  LOCATE ln%, col%
  PRINT SPACE$(ic%) 'clear input zone
END DO

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ELSEIF (ASC(ch$) = 8) THEN  
  '<BACKSPACE> delete last  
  'position on erroneous character  
  'print a space  
  'decrement character counter  
  LOCATE ln%, col% + ic% - 1  
  'increment character counter  
  PRINT ch$;  
  'print the new character  
  'add to the title  
  title$ = title$ + ch$  
  END IF  
  LOOP UNTIL (ic% = tl%)  
  title$ = LTRIM$(title$)  
  'delete any leading spaces  
  LOCATE ln%, col%  
  'clear the display  
  PRINT title$  
  'echo the title  
  stitle$ = title$  
  END FUNCTION

SUB cenprt (ln%, btitle$)  
  'Centres and displays a frame title.  
  l% = LEN(btitle$)  
  c1% = (BO - 1%) \ 2  
  LOCATE ln%, cl%; PRINT btitle$;  
  END SUB

SUB choose (mn%, items%, key$, sel%)  
  'Allows a menu item to be selected.  
  key$ = readkey$  
  SELECT CASE key$  
  CASE CHR$(27)  
    sel% = 0  
  CASE CHR$(0) + "G"  
    'home key  
    sel% = 1  
  CASE CHR$(0) + "O"  
    'end key  
    sel% = items%  
  CASE CHR$(0) + "M"  
    'up arrow  
    sel% = sel% - 1  
    IF (sel% = 0) THEN sel% = items%  
  CASE CHR$(0) + "D"  
    'down arrow  
    sel% = sel% + 1  
    IF (sel% = items% + 1) THEN sel% = 1  
  END SELECT  
  END SUB

FUNCTION datemod$  
  'Changes the internal date format (mm-dd-yyyy) to (dd mon YYYY).  
  mon$ = LEFT$(DATES, 2)  
  'month  
  nmon% = VAL(mcn$)  
  mon$ = mon(nmon%)  
  yr$ = RIGHT$(DATES, 4)  
  'year  
  day$ = MIDS(DATE$, 4, 2)  
  'day  
  IF (LEFT$(day$, 1) = "0") THEN  
    days = RIGHTS(day$, 1)  
  END IF  
  datemod$ = day$ + " " + mon$ + " " + yr$  
  'combine  
  END FUNCTION

FUNCTION datval! (vnum$, iok%)  
  'Accepts a string representing a number, checks its validity  
  'and returns the number, with:  
  'vnum$ = the string; vnum = the required number;  
  'iok% = 0 if the string is invalid, otherwise iok% = 1.  
  iok% = 0  
  vnum$ = LTRIM$(vnum$)  
  'assume string is invalid  
  j% = LEN(vnum$)  
  'length of string  
  IF (j% = 0) THEN  
    'null string  

EXIT FUNCTION 'exit

'first, check for any invalid characters.
ELSE
FOR i% = 1 TO j%
  ch$ = MID$(vnum$, i%, 1)
  k% = ASC(ch$)
  IF (k% > 47 AND k% < 58) THEN
    ELSEIF (k% = 46) THEN
      ELSEIF (k% = 43 OR k% = 45) THEN
        ELSEIF (k% = 69 OR k% = 101) THEN
          ELSE
            EXIT FUNCTION
        END IF
      NEXT i%
'Now check first character.
  ch$ = LEFT$(vnum$, 1)
  k% = ASC(ch$)
  IF (k% = 69 OR k% = 101) THEN
    EXIT FUNCTION
  END IF
'Count the occurrences of "E/e", "+/-" and ".".
  ec% = 0: sc% = 0: pc% = 0 'zero counters
  FOR i% = 1 TO j%
    ch$ = MID$(vnum$, i%, 1)
    k% = ASC(ch$)
    IF (i% > 1) THEN
      ch$ = MID$(vnum$, i% - 1, 1)
      kk% = ASC(ch$)
    END IF
    IF (k% = 69 OR k% = 101) THEN
      ec% = ec% + 1
      IF (ec% = 2) THEN
        EXIT FUNCTION
      END IF
    ELSEIF (k% = 43 OR k% = 45) THEN
      sc% = sc% + 1
      IF (sc% = 3) THEN
        EXIT FUNCTION
      END IF
    ELSEIF (k% = 46) THEN 'extract a character
      ELSE
        EXIT FUNCTION
    ELSE
      EXIT FUNCTION
    END IF
    NEXT i%
  IF (k% = 69 OR k% = 101) THEN 'the previous character
    EXIT FUNCTION
  END IF
  ELSEIF (k% = 46) THEN
    sc% = sc% + 1
    IF (sc% = 3) THEN
      EXIT FUNCTION
    END IF
    ELSEIF (i% <> 1) THEN
      IF (kk% <> 69 AND kk% <> 101) THEN
        EXIT FUNCTION
      END IF
    ELSEIF (k% = 43 OR k% = 45) THEN
      ELSE
        EXIT FUNCTION
    ELSE
      EXIT FUNCTION
    END IF
    NEXT i%
  iok% = 1 'OK to convert
  vnum = VAL(vnum$)
  datval! = vnum
END FUNCTION

SUB delay (dt)
'Provides a delay of dt seconds.
  t1 = TIMER
  DO
    t2 = TIMER
    LOOP UNTIL (t2 > (t1 + dt))
END SUB

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FUNCTION drive$(ln%, col\%)

' Returns a disc drive letter, A, B, C or D.

DO
  LOCATE ln%, col\%
  PRINT SPACE$(3)
  a$ = INPUT$(1)
  a$ = UCASE$(a$)
  LOOP UNTIL (ASC(a$) > 64 AND ASC(a$) < 69)
  LOCATE ln%, col\%
  PRINT a$
  drive$ = a$
END FUNCTION

SUB FileOpen (ext$, flnm$)

' Opens a file after requesting the drive and directory/sub-directory.

CLS
CALL headframe(1, 1, 80, 25)
btitle$ = "Data file Name"

DO
  CALL headframe(7, 18, 44, 11)
  CALL cenprt(7, btitle$)
  LOCATE 8, 20
  PRINT "Choose the disc drive (A, B, C or D): ";
  dd$ = drive$(8, 56) ' disc drive
  LOCATE 9, 20
  PRINT "Name the directory\sub-directory: ",
  LOCATE 10, 24
  INPUT ">", ddir$ 'directory
  LOCATE 11, 20
  PRINT "Data file (max.of 8 ch.),:";
  LOCATE 12, 24
  PRINT USING "the current extension \" will be appended: "; ext$;
  filename$ = atitle$(13, 25, 8)
  a$ = affirm$(14, 20, "Correct")
  LOOP UNTIL (a$ = "y")
  location$ = dd$ + ":\"
  IF (ddir$ <> ") THEN location$ = location$ + ddir$ + "\"
  flnm$ = location$ + filename$ + "." + ext$ ' add extension
  OPEN flnm$ FOR OUTPUT AS #1 ' open file
  LOCATE 15, 20
  PRINT "Data file open: ", flnm$
  LOCATE 17, 30
  PRINT "Any key to continue ", a$ = readkey$
END SUB

SUB headframe (tr\%, lc\%, wd\%, dp\%)

' Draws a simple box around headings.

  LOCATE tr\%, lc\%
  PRINT CHR$(218); STRING$(wd\% - 2, 196); CHR$(191); ' top
  ln% = tr%
  FOR i% = 1 TO dp\% - 2
    ln% = ln% + 1
    LOCATE ln%, lc%
    PRINT CHR$(179); SPACES(wd\% - 2); CHR$(179);
  NEXT i%
  LOCATE tr\% + dp\% - 1, lc%
  PRINT CHR$(192); STRING$(wd\% - 2, 196); CHR$(217); ' bottom
END SUB
SUB lines (k%)  
'Sends blank lines to the print file.  
FOR i% = 1 TO k%  
PRINT $2, ""  
NEXT i%  
END SUB

SUB menu (btitle$, tr%, lc%, sel%)  
'Reads required menu list, displays the menu and allows selection.

items% = 0: maxl% = 0  
FOR i% = 1 TO 10  
1% = LEN(item$(i%))  
IF (1% > maxl%) THEN maxl% = 1%  
END IF  
NEXT i%  

1% = LEN(btitle$)  
IF (1% > maxl%) THEN maxl% = 1%  
ELSE  
items% = i%  
END IF  

dp% = items% + 1  
rd% = maxl% + 3  
CALL headframe(tr%, lc%, wd%, dp%)  
CALL cenprt(tr%, btitle$)  
11% = wd% - 1  

DO  
FOR i% = 1 TO items%  
IF (i% = sel%) THEN  
COLOR 14  
ELSE  
COLOR 0  
END IF  
1% = LEN(item$(i%))  
lspl% = (11% - 1%) \ 2  
lspr% = 11% - 1% - lspl%  
line$ = SPACE$(1spl%) + item$(i%) + SPACE$(1spr%)  
LOCATE tr% + i%, lc% + 1  
PRINT line$  
NEXT i%  

CALL choose(mn%, items%, key$, sel%)  
LOOP UNTIL (key$ = CHR$(13) OR key$ = CHR$(27))  
CALL headframe(tr%, lc%, wd%, dp%)  
END SUB

FUNCTION ok$  
'Returns a 'y' or 'n' answer.  
as$ = INPUT$(1)  
as$ = LCASE$(a$)  
PRINT a$  
ok$ = a$  
END FUNCTION

SUB openscr (line1$, line2$, line3$, line4$, name$)  
'The title screen.

CLS  
COLOR 14, 1  
CALL headframe(1, 1, 80, 25)  
LOCATE 3, 10: PRINT "RuPEC";  
LOCATE 9, 13: PRINT line1$;  
LOCATE 11, 13: PRINT line2$;
LOCATE 13, 13: PRINT line3$;
LOCATE 15, 13: PRINT line4$;
LOCATE 23, 10: PRINT name$;
CALL delay(2)
END SUB

FUNCTION readkey$
',
'Accepts a single key.
',
WHILE INKEY$ <> "": WEND
'clear keyboard buffer
DO UNTIL (a$ <> "")
'wait for key
a$ = INKEY$
LOOP
'read key
readkey$ = a$
END FUNCTION
Appendix C

TNRHOUN.BAS

---

**Program "TNRHOUN.BAS"**

Based on the Turner model of elasticity, the tension in the diagonal elastic member is represented by the original 4 parameter function or a polynomial up to order 5. Using the raw Hounsfield test data: uniaxial engineering stress versus percentage strain (both in coded form); the 4 parameters are fitted by scanning and adjustment; whilst the polynomial coefficients are fitted by multiple regression analysis and adjustment.

The goodness of the fit is displayed as a selection of graphs.

The coefficients are written to a file with the extension ".tnl" or "tn2", appended to which are failure results and a table of stress and strain.


Based upon tprops.bas and hountrn.bas.

---

DECLARE SUB assem (nt%)
DECLARE SUB assess (title$, subtitle$, tdate$)
DECLARE SUB calcs (lam$, stress)
DECLARE SUB calcurv (k%, ymin, xsc, ysc)
DECLARE SUB cenprt (ln%, btitle$)
DECLARE SUB change (alter%)外面不知情
DECLARE SUB check (j%)
DECLARE SUB chgraph ()
DECLARE SUB chmodconsts (nt%, title$, inst)
DECLARE SUB choose ()
DECLARE SUB chorigconsts (title$, inst)
DECLARE SUB control (nt%, title$, a$)
DECLARE SUB create (nt%)
DECLARE SUB dataform ()
DECLARE SUB dataselect (j%, xy())
DECLARE SUB datasumm (title$, subtitle$, tdate$)
DECLARE SUB decide (dec$)
DECLARE SUB drawpts (k%, d%, title$)
DECLARE SUB exppts (d%, k%, ymin, xsc, ysc)
DECLARE SUB failsave ()
DECLARE SUB fopen (f%, flnm$)
DECLARE SUB forcext (title$)
DECLARE SUB graphtitle ()
DECLARE SUB headframe (tr%, lc%, wd%, dp%)
DECLARE SUB highlow (j%, yhigh, ylow)
DECLARE SUB lambdas (l%)
DECLARE SUB legend (d%, k%)
DECLARE SUB listres (name$)
DECLARE SUB mark (j%, xp%, yp%)
DECLARE SUB minresult (fi%, fj%, fk%, fl%)
DECLARE SUB modequation (param%)
DECLARE SUB modified (title$, dec%, a$)
DECLARE SUB openscr ()
DECLARE SUB origequation (param%)
DECLARE SUB original (title$, a$)
DECLARE SUB parameters ()
DECLARE SUB prtselect (j%)
DECLARE SUB ranges ()
DECLARE SUB readdata ()
DECLARE SUB saveres (name$, newdate$)
DECLARE SUB scatter (nt%)
DECLARE SUB scrttitle ()
DECLARE SUB solv (nt%)
DECLARE SUB sweep (fi%, fj%, fk%, fl%)

---

Page C1
DECLARE SUB terms()
DECLARE SUB wththk(j%)
DECLARE SUB work(sumlam, sed)

DECLARE FUNCTION affirm$ (ln%, col%, text$)
DECLARE FUNCTION alog! (x)
DECLARE FUNCTION atitle$ (ln%, col%, tl%)
DECLARE FUNCTION cosine! (j%, I%)
DECLARE FUNCTION datemod$( )
DECLARE FUNCTION datval! (vnum$, iok%)
DECLARE FUNCTION delay! (dt)
DECLARE FUNCTION drive$ (ln%, col%)
DECLARE FUNCTION engstress! (j%, yval)
DECLARE FUNCTION lambda! (x)
DECLARE FUNCTION ok$ ()
DECLARE FUNCTION readkey$ ()
DECLARE FUNCTION strain! (j%, I%)

DIM SHARED coeff(5) AS STRING * 1, mon(12) AS STRING * 3
DIM SHARED ndp(9), sndp(9), term(9) AS INTEGER
DIM SHARED wth(9), thk(9) AS SINGLE
DIM SHARED lam(9, 25), feng(9, 25), ftrue(9, 25) AS SINGLE
DIM SHARED lammx(9), fengmx(9), ftruemx(9) AS SINGLE
DIM SHARED lamsig(2, 25) AS SINGLE
DIM SHARED x(6, 250), sum(6), a(6, 6), bb(5), c(6) AS SINGLE

COMMON SHARED filename AS STRING
COMMON SHARED samples, sndp, tndp, gr, lq, tnr, ms, logsq, npr AS INTEGER
COMMON SHARED dataf AS INTEGER
COMMON SHARED loading, extrng, speed, lammax, sd AS SINGLE
COMMON SHARED pretension, econst1, econst2, nnp AS SINGLE

CONST false = 0, true = NOT false

REM $DYNAMIC

on error go to message

consts: DATA "b", "c", "d", "e", "f"

months: DATA "Jan", "Feb", "Mar", "Apr", "May", "Jun"
        DATA "Jul", "Aug", "Sep", "Oct", "Nov", "Dec"

RESTORE consts
FOR I% = 1 TO 5
  READ coeff(I%)
NEXT I%

RESTORE months
FOR I% = 1 TO 12
  READ mon(I%)
NEXT I%

newdate$ = datemod$

FOR I% = 1 TO 9
  sndp(I%) = 25
NEXT I%

CALL scrtile
SCREEN 2
SCREEN 0
COLOR 7, 1

CALL dataform
CALL fopen(1, flnm$)
 IF (dataf = 1) THEN
   CALL assess(title$, subtitle$, tdate$) 'assesses data in file
   CALL datasumm(title$, subtitle$, tdate$) 'summarises data read
   CALL fopen(2, flnm$) 'reopen data file
 END IF

 IF (dataf = 1) THEN
   CALL readdata 'read data, select and 'convert
 ELSE
   CALL forcext(title$) 'force-extension data
 END IF
Appendix C

FOR I% = 1 TO samples
chd(I%) = 1
NEXT I%

'all samples
'to be included initially

1g = false: gr = 1
CALL drawpts(1, 1, titleS)
= affirmS(28, 5, "Screen dump")
IF (a$ = "y") THEN
CALL drawpts(2, 1, title$)
COLOR 15
LOCATE 30, 5
PRINT " Press <Shift><Prt Sc> ";
LOCATE 30, 56
PRINT " Any key to continue ";
aS = readkey$
END IF
SCREEN 0
COLOR 7, 1

'graph data points only
'for screen dump

're-establish text screen
'best fit loop
'how to proceed
'terminate

DO
CALL decide(dec%)
IF (dec% = -1) THEN
COLOR 7, 0
CLS
END
END IF
IF (dec% = 1 OR dec% = 3) THEN
CALL choose
END IF
IF (tnr = 1) THEN
CALL original(titleS, a$)
ELSEIF (tnr = 2) THEN
CALL modified(titleS, dec%,
END IF

'choose samples

s,$)

LOOP UNTIL (a$ = "y")

'best fit chosen

CALL drawpts(1, 2, titleS)
aS = affirmS(28, 5, "Screen dump")
IF (aS = "y") THEN
CALL drawpts(2, 2, titleS)
COLOR 15
LOCATE 30, 5
PRINT " Press <Shift><Prt Sc>
LOCATE 30, 56
PRINT " Any key to continue ";
a$ = readkey$
END IF
SCREEN 0
COLOR 7, 1

'for screen dump

're-establish text screen

CALL listres(titleS)

'list the results

CALL fopen(3, flnmS)
CALL saveres(titleS, newdate$)

'open results file
'write the result to file

COLOR 7, 0
CLS
END
'Error messages.
1

message:
SCREEN 0
COLOR 7, 1
CLS
LOCATE 8, 30
PRINT "AN ERROR HAS OCCURRED."
LOCATE 13, 10
PRINT "Note any displayed message:"
LOCATE 15, 15
SELECT CASE ERR
CASE 5
PRINT "An illegal function call has been made."
CASE 6
PRINT "Overflow has occured,"
LOCATE 16, 20
PRINT "- a variable value is too large."

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CASE 9
  PRINT "An array subscript is out of its allowable range."
CASE 11
  PRINT "An attempt has been made to divide by zero."
CASE 27
  PRINT "The printer is not on line."
CASE 52, 53, 54, 55, 56
  PRINT "A file problem has been encountered."
CASE 57
  PRINT "An I/O fatal error has occurred."
CASE 61
  PRINT "The disc receiving output is full."
CASE 71, 72
  PRINT "There is no disc or it is flawed."
CASE ELSE
  PRINT "An unidentified error has occurred."
END SELECT

LOCATE 20, 10
PRINT "Any key to quit."
INPUT $ = readkey$
COLOR 7, 0
CLS
END

REM $STATIC

----SUBROUTINES----

FUNCTION affirm$ (ln%, col%, text$)
  'Accepts a question and returns a "y/n" answer.
  'See RUPEC.BAS
  END FUNCTION

FUNCTION alog! (x)
  'Changes the base of a logarithm from "e" to 10.
      xx = LOG(x) / LOG(10)
      alog! = xx
END FUNCTION

SUB assem (nt%)
  'Assembles equations for multiple regression.
      DIM sumsq(6, 6)
      FOR I% = 1 TO nt%  'zero summing arrays
          sum(I%) = 0
      FOR j% = 1 TO nt%
          sumsq(I%, j%) = 0
          NEXT j%
      NEXT I%

      FOR j% = 1 TO nt%  'sums
          FOR I% = 1 TO tndp
              sum(j%) = sum(j%) + x(j%, I%)
          NEXT I%
      NEXT j%

      FOR j% = 1 TO nt%
          FOR k% = 1 TO nt%
              FOR I% = 1 TO tndp
                  sumsq(j%, k%) = sumsq(j%, k%) + x(j%, I%) * x(k%, I%)
              NEXT I%
          NEXT k%
      NEXT j%

      'Form matrix (including rhs), then extract r.h.s. vector.
          FOR j% = 1 TO nt%
              FOR k% = 1 TO nt%
                  FOR I% = 1 TO tndp
                      a(j%, k%) = sumsq(j%, k%) + sum(j%) * sum(k%) / tndp
                  NEXT I%
              NEXT k%
          NEXT j%
Appendix C

FOR j% = 1 TO nt% - 1
  c[1]<j%> = a[1]<j%, nt%>
NEXT j%
END SUB

SUB assess (title$, subtitle$, tdate$)
  "Reads the data file to determine the output file name, the title, the number of samples, the number of data points recorded for each sample, the load range and the extension range.
  "
  INPUT #1, dum$  'unwanted text
  INPUT #1, filename  'to be used for output file
  INPUT #1, subtitle$  'subtitle
  INPUT #1, title$  'main title
  INPUT #1, tdate$  'test date
  FOR I% = 1 TO 5
    'unwanted data
  NEXT I%
  INPUT #1, dum$, loadrng  'load range
  INPUT #1, dum$, extrng  'extension range
  INPUT #1, dum$, ndum  'unwanted data
  INPUT #1, dum$, speed  'test speed
  samples = 0  'zero number of samples
  DO UNTIL (EOF(1))  'read file
    LINE INPUT #1, line$
    IF (MID$(line$, 2, 4) = "Plot") THEN  'sample found
      samples = samples + 1
      IF (samples > 9) THEN
        CLS
        CALL cerprt(5, "There are more than 9 samples.")
        CALL cerprt(10, "PROGRAM ABANDONED!")
        END END IF
    END IF
    ndp(samples) = 0  'zero number of data points
    xx = 0  'ignore "," in numbers
    DO UNTIL (xx = -1)
      INPUT #1, xx, yy
      INPUT #1, ndp(samples) + 1  'input data
      LOOP
      ndp(samples) = ndp(samples) - 1  'count data points
      IF (ndp(samples) > 1500) THEN
        CLS
        txt$ = "Sample" + STR$(samples) + " has more than 1500 data points.")
        CALL cerprt(10, "PROGRAM ABANDONED!")
        END END IF
    END IF
  END IF
  END SUB

FUNCTION atitle$ (ln%, col%, tl%)
  'Enters a title with a maximum number of characters, with:
  '  ln% = line;  col% = column of first character entered;
  '  tl% = maximum length of the title.
  'Note: > and < are displayed at either end of the input zone.
  'See RUPEC.BAS
END FUNCTION

SUB calcs (laml, stress)
  'Calculates the uniaxial engineering stress using the fitted parameters.
  lengo = SQR(3)  'original diagonal length
  lam2 = 1 / SQR(laml)
  lam3 = lam2
  length = SQR(laml * lam2 + lam2 + lam3 + lam3)  'diagonal length
  extn = (length - lengo) / lengo  'diagonal strain
  alf1 = ATN(SQR(lam2 * lam3 + lam3 + lam2 + lam3) / laml)  'angles
  alf2 = ATN(SQR(lam3 * lam2 + lam3 + lam2 + lam3) / lam2)
  alf3 = ATN(SQR(laml * lam2 + lam1 + lam2 + lam2) / lam3)
  IF (tnr = 1) THEN  'original fn.
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\[
tens = \text{pretension} + \text{econst1} \cdot \text{extn} + \text{econst2} \cdot \text{extn} \cdot \text{nnp}
\]

ELSEIF (tnr = 2) THEN
\[
tens = \text{bb}(0)
\]
\[
k\% = 0
\]
FOR \(j\% = 1\) TO 5
\[
\text{IF (term}(j\%)\text{) = 1) THEN}
\]
\[
k\% = k\% + 1
\]
\[
tens = tens + \text{bb}(k\%) \cdot \text{extn} \cdot j\%
\]
END IF
NEXT \(j\%\)
END IF

\[
\text{forcl = tens} \cdot \text{COS}(\text{alf1})
\]
\[
\text{forc2 = tens} \cdot \text{COS}(\text{alf2})
\]
\[
\text{forc3 = tens} \cdot \text{COS}(\text{alf3})
\]
\[
\text{strs1 = forcl} \cdot \text{laml}
\]
\[
\text{strs2 = forc2} \cdot \text{lamm2}
\]
\[
\text{strs3 = forc3} \cdot \text{lamm3}
\]
\'
\[
\text{Correct true stress for hydrostatic pressure.}
\]
\[
\text{stress = strs1 - strs3}
\]
END SUB

SUB calcurv (k\%, ymin, xsc, ysc)
\'
\[
\text{Draws the calculated stress-strain curve.}
\]
\[
col\% = 14
\]
\[
\text{IF (k\% = 2) THEN col\% = 15}
\]
\[
\text{IF (lg) THEN}
\]
FOR I\% = 1 TO 25
\[
\text{xp\% = 200 + CINT(}(\text{lamsig}(1, I\%) \cdot - 1) \cdot xsc\text{)}
\]
SELECT CASE gr
CASE 1
\[
ydiff = \text{alogn}(\text{lamsig}(2, I\%)) - \text{ymin}
\]
CASE 2
\[
ydiff = \text{alogn}(\text{lamsig}(2, I\%) / \text{lamsig}(1, I\%)) - \text{ymin}
\]
CASE 3
\[
ydiff = \text{alogn}(\text{lamsig}(2, I\%) / (\text{lamsig}(1, I\%) - 1)) - \text{ymin}
\]
CASE 4
\[
\text{engstr = lamsig}(2, I\%) \cdot \text{lamsig}(1, I\%)
\]
\[
ydiff = \text{alogn}(\text{engstr} / (\text{lamsig}(1, I\%) - 1)) - \text{ymin}
\]
END SELECT
\[
\text{yp\% = 352 - CINT(ydiff \cdot ysc)}
\]
\[
\text{IF (I\% = 1) THEN}
\]
PSET (xp\%, yp\%)
ELSE
LINE -(xp\%, yp\%), col\%
END IF
NEXT I\%
ELSE
\[
\text{IF (gr = 1 OR gr = 2) THEN}
\]
xp\% = 200: yp\% = 352
PSET (xp\%, yp\%), col\%
END IF
FOR I\% = 1 TO 25
\[
\text{xp\% = 200 + CINT((lamsig}(1, I\%) \cdot - 1) \cdot xsc\text{)}
\]
SELECT CASE gr
CASE 1
\[
\text{yp\% = 352 - CINT(ydiff} \cdot ysc\text{)}
\]
CASE 2
\[
\text{yp\% = 352 - CINT(ydiff} \cdot ysc / \text{lamsig}(1, I\%)\text{)}
\]
CASE 3
\[
\text{yp\% = 352 - CINT(ydiff} \cdot ysc / (\text{lamsig}(1, I\%) - 1))
\]
CASE 4
\[
\text{engstr = lamsig}(2, I\%) \cdot \text{lamsig}(1, I\%)
\]
\[
\text{yp\% = 352 - CINT(engstr} \cdot ysc / (\text{lamsig}(1, I\%) - 1))
\]
END SELECT
\[
\text{IF (I\% = 1 AND (gr = 3 OR gr = 4)) THEN}
\]
PSET (xp\%, yp\%)
ELSE
LINE -(xp\%, yp\%), col\%
END IF
NEXT I\%
END IF
END SUB
SUB cenprt (ln%, btitle$)
'Centres and displays text.
'See RUPEC.BAS

END SUB

SUB change (alter$)
'Returns a percentage change according to the key pressed.

DO
a$ = readkey$
SELECT CASE a$
CASE CHR$(27)
alter% = -100
CASE CHR$(0) + "H"
alter% = 1
CASE CHR$(0) + "G"
alter% = 10
CASE CHR$(0) + "P"
alter% = -1
CASE CHR$(0) + "O"
alter% = -10
CASE CHR$(48)
alter% = 0
CASE CHR$(45)
alter% = -200
CASE ELSE
alter% = 100
 END SELECT
LOOP UNTIL (alter% < 20)
END SUB

SUB check (j%)
'Checks the data for increasing lambda and stress.

lchk% = 0: fchk% = 0
FOR I% = 2 TO sndp(j%)
IF (lam(j%, I%) < lam(j%, I% - 1)) THEN
lchk% = 1
1% = I% - 1; 12% = I%
END IF
IF (feng(j%, I%) < feng(j%, I% - 1)) THEN
fchk% = 1
13% = I% - 1; 14% = I%
END IF
NEXT I%
LOCATE 21, 10
IF (lchk% = 1) THEN
PRINT USING "WARNING: lambda## exceeds lambda#0."; 11%; 12%;
ELSE
PRINT "CHECK: the lambda values increase in sequence."
END IF
LOCATE 22, 10
IF (fchk% = 1) THEN
PRINT USING "WARNING: stress## exceeds stress##"; 13%; 14%;
ELSE
PRINT "CHECK: the stress values increase in sequence."
END IF
END SUB

SUB chgraph
'Allows various graphs to be selected.

SCREEN 0
COLOR 7, 1
CLS
CALL headframe(1, 1, 80, 25)
DO
CALL headframe(5, 18, 44, 11)
CALL cenprt(5, " GRAPH CHOICE ")
LOCATE 6, 20: PRINT "Options:"
LOCATE 7, 25: PRINT "0. Quit graphs;"

Appendix C

LOCATE 8, 25: PRINT "1. True stress v. lambda;"
LOCATE 9, 25: PRINT "2. Engineering stress v. lambda;"
LOCATE 10, 25: PRINT "3. True secant modulus v. lambda;"
LOCATE 11, 25: PRINT "4. Eng. secant modulus v. lambda;"
DO
  LOCATE 12, 20: PRINT SPACE$(36)
  LOCATE 12, 20: PRINT "Choose 1 to 4: ";
  ch$ = INPUT$(1): PRINT ch$
  ch = datval!(ch$, iok%)
  qr = CINT(ch)
  LOOP UNTIL (iok% = 1 AND (qr >= 0 AND qr < 5))
  lg = false
  a$ = affirm$(13, 20, "Logarithmic ordinate")
  IF (a$ = "y") THEN lg = true
  a$ = affirm$(14, 20, "Correct")
  LOOP UNTIL (a$ = "y")
END SUB

SUB chmodconsts (nt%, title$, inst)
'Allows the user to modify each of the Turner polynomial coefficients.
  DO 'main modification loop
    param% = -1
    CALL modequation(param%)
    'display polynomial function
    LOCATE 26, 5: PRINT SPACE$(36):
    LOCATE 26, 5: PRINT "Enter selection (ESC to exit): ";
    DO
      a$ = INPUT$(1)
      a$ = LCASE$(a$)
      IF (a$ = CHR$(27)) THEN 'ESC to exit and recalculate
        EXIT SUB 'curve
      ELSE 'check choice
        param% = ASC(a$) - 97 'parameter index (0 to 5)
        cok% = 0 'set flag for acceptance
        IF (param% > -1 AND param% < 6) THEN 'acceptable parameter
          cok% = 1 'flag
          LOCATE 26, 36
          PRINT " ";
          PRINT a$: 'echo choice
          END IF
        END IF
      END IF
    END IF
    CALL modequation(param%)
    'equation with highlight
    k% = 0 'find associated coeff.
    FOR I% = 1 TO param%
      IF (term(I%) = 1) THEN k% = k% + 1
    NEXT I%
    'Use arrow keys to alter chosen constant.
    DO
      fmt$ = "Current value = #.##"^(0)
      LOCATE 26, 5: PRINT SPACE$(32):
      LOCATE 26, 5: PRINT USING fmt$: bb(k%):
      'associated coefficient
      txt$ = "Press: " + CHR$(24) + " (+1%) " + CHR$(25) + " (-1%)"
      LOCATE 27, 5
      PRINT txt$
      txt$ = " HOME (+10%) END (-10%) or ESC"
      LOCATE 28, 5
      PRINT txt$
      CALL change(alter%)
      'key press
    'Alter chosen coefficient up or down 1% or 10%.
    IF (alter% = -100) THEN
      EXIT DO
    ELSE
      IF alter% = 0 THEN
        bb(k%) = 0 'set coef to zero
      ELSE

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IF bb(k%) = 0 THEN bb(k%) = 1  'set to one if previously zeroed
chcoeff = alter% * .01  'set size and sign
bb(k%) = bb(k%) + chcoeff * bb(k%)  'alter non zero coefficient
END IF
IF (inst) THEN  'instant display
FOR I% = 1 TO 25
   lamsig(1, I%) = 1 + I% + (lammax - 1) / 25
   lam1 = lamsig(1, I%)
   CALL calcs(lam1, stress)  'calculate fitted stress
   lamsig(2, I%) = stress
NEXT I%
CALL scatter(nt%)  'eng.stress residual S.D.
CALL drawpts(1, 2, title$)  'draw graph
CALL modequation(param%)  'equation with highlight
END IF
END IF
END LOOP
END LOOP
END
SUB
SUB
choose
'Allows samples to be ignored.
'See RUPEC.BAS
END SUB
SUB chorigconsts {title$, inst}
'Using sub "drawpts", allows the user to modify each of the Turner constants.
'END SUB
DO  'main modification loop
   param% = 0
   CALL origequation(param%)
   LOCATE 26, 5: PRINT SPACE$(36); LOCATE 26, 5: PRINT "Enter selection (ESC to exit): ";
   DO
      a$ = INPUT$(1)
      a$ = LCASE$(a$)
      IF (a$ = CHR$(27)) THEN  'ESC to exit
         EXIT
      END IF
      COLOR 12
      SELECT CASE a$
      CASE "a", "A"
         param% = 1
         LOCATE 25, 5: PRINT "A";
      CASE "b", "B"
         param% = 2
         LOCATE 25, 9: PRINT "B";
      CASE "c", "C"
         param% = 3
         LOCATE 25, 13: PRINT "C";
      CASE "n", "N"
         param% = 4
         LOCATE 25, 19: PRINT "n";
      CASE ELSE
         param% = -1
      END SELECT
      LOOP UNTIL (param% <> -1)
      COLOR 14
      'Use arrow keys to alter chosen constant.
      DO
         CALL origequation(param%)
         fmt$ = "Current value =fli.#fil"
         LOCATE 26, 5: PRINT SPACE$(32); LOCATE 26, 5
         SELECT CASE param%
         CASE 1
            PRINT USING fmt$; pretension;
         CASE 2
         END SELECT
      END DO
   END DO
   CALL origequation(param%)
END SUB
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PRINT USING fmt$; econst1;  'linear coeff.
CASE 3
PRINT USING fmt$; econst2;  'power coeff.
CASE 4
PRINT USING fmt$; nnp;  'power index
END SELECT

txt$ = "Press: " + CHR$(24) + " (+1%) " + CHR$(25) + " (-1%)"
LOCATE 27, 5
PRINT txt$;

CALL change(alter%)
  'select key

'Alter constants up or down 1% or 10%.
  IF (alter% = -100) THEN
    EXIT DO
  ELSE
    mult = alter% * .01  'set size and sign
    SELECT CASE param%
    CASE 1
      IF alter% = 0 THEN
        pretension = 0
      ELSE
        IF pretension = 0 THEN pretension = 1
       pretension = pretension + (mult * pretension)
      END IF
    CASE 2
      IF alter% = 0 THEN
        econst1 = 0
      ELSE
        IF econst1 = 0 THEN econst1 = 1
        econst1 = econst1 + (mult * econst1)
      END IF
    CASE 3
      IF alter% = 0 THEN
        econst2 = 0
      ELSE
        IF econst2 = 0 THEN econst2 = 1
        econst2 = econst2 + (mult * econst2)
      END IF
    CASE 4  'power does not need to be 0
      nnp = nnp + (mult * nnp)
    END SELECT

'REcalculate curve.
  IF (inst) THEN  'instant display
    FOR I% = 1 TO 25
      lamsig(1, I%) = 1 + I% * (lammax - 1) / 25
      laml = lamsig(1, I%)
      CALL calcs(laml, stress)  'calculate fitted stress
      lamsig(2, I%) = stress
    NEXT I%
    CALL scatter(nt%)  'calculate the residual S.D.
    CALL drawpts(1, 2, title$)  'draw graph
    CALL origequation(param%)  'equation with highlight
  END IF
END IF
LOOP
LOOP
END SUB

SUB control (nt%, title$, a$)
  'Controls the graphing and adjustment of the coefficients.
  CALL chgraph  'graph type
  IF (gr = 0) THEN
    a$ = "n"  'quit
    EXIT SUB
  END IF
  DO  'parameter adjustment
    FOR I% = 1 TO 25
      lamsig(1, I%) = 1 + I% * (lammax - 1) / 25
      laml = lamsig(1, I%)
    NEXT I%
  END IF
END SUB
CALL calcs(lambda1, stress)  
  
  calcsig(2, I%) = stress  
  
  NEXT I%  
  CALL scatter(nt%)  
  CALL drawpts(1, 2, title$)  
  
  a$ = affirm(27, 5, "Adjust coeff.s")  
  IF (a$ = "y") THEN  
    a$ = affirm(28, 5, "Change graph")  
    CALL chgraph  
    CALL drawpts(1, 2, title$)  
  END IF  
  
  IF (a$ = "y") THEN  
    CALL chorigconsts(title$, inst)  
    ELSEIF (tnr = 1) THEN  
      CALL chmodconsts(nt%, title$, inst)  
    END IF  
  ELSE  
    EXIT DO  
  END IF  
  
  LOOP  
  
  'Calculate stress
  lamsig(2, I%) = stress  
  
  NEXT I%  
  CALL scatter(nt%)  
  CALL drawpts(1, 2, title$)  
  
  a$ = affirm(27, 5, "Adjust coeff.s")  
  IF (a$ = "y") THEN  
    a$ = affirm(28, 5, "Change graph")  
    CALL chgraph  
    CALL drawpts(1, 2, title$)  
  END IF  
  
  IF (gr > 0) THEN  
    dump% = 0  
    CALL drawpts(2, 2, title$)  
    aa$ = affirm(27, 5, "Screen dump")  
    IF (aa$ = "y") THEN  
      dump% = 1  
      CALL drawpts(2, 2, title$)  
      COLOR 15  
      LOCATE 30, 4  
      PRINT " Press <Shift><Prt Sc> ";  
      LOCATE 30, 56  
      PRINT " Any key to continue ";  
      aa$ = readkeys$  
    END IF  
  END IF  
  
  LOOP UNTIL (a$ = "n")  
  CALL drawpts(1, 2, title$)  
  
  a$ = affirm(28, 5, "Best fit")  
  END SUB

FUNCTION cosine!(j%, I%)  
  
  Calculates the angle of the elastic member to lambda.  
  
  cs = 1 / SQR(1 + 2 / lam(j%, I%) ^ 3)  
  cosine! = cs  
  END FUNCTION

SUB create(nt%)  
  
  'Creates the terms for multiple regression, where nt% is the number  
  'of terms included.  
  
  tndp = 0  
  FOR k% = 1 TO samples  
    IF (chd(s%) = 1) THEN  
      FOR I% = 1 TO sndp(s%)  
        k% = 0  
        FOR j% = 1 TO 5  
          IF (term(j%) = 1) THEN  
            k% = k% + 1  
            x(k%, tndp + I%) = (str(s%, I%) ^ j%) * cosa(s%, I%)  
          END IF  
        END FOR  
      END FOR  
    END IF  
  END FOR  
  END SUB

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NEXT j%

NEXT I%

FOR I% = 1 TO sndp(s%)
  x(nt% + 1, tndp + I%) = feng(s%, I%)
NEXT I%

tpd = tndp + sndp(s%)
END IF

NEXT s%
END SUB

SUB dataform
  'A choice is made between a "*.raw" file or a "*.fex" file, the latter
  being values of force (N) and extension (mm).

  CLS
  CALL headframe(1, 1, 80, 25)
  CALL headframe(5, 23, 34, 6)
  CALL centprt(5, " FORM OF DATA ")

  LOCATE 6, 25
  PRINT "Choice:
  LOCATE 7, 30
  PRINT "1. Raw Hounsfield data;"
  LOCATE 8, 30
  PRINT "2. Force-extension data."

  DO
    LOCATE 9, 25
    PRINT SPACE$(30)
    LOCATE 9, 25
    PRINT "Select 1 or 2: ";
    a$ = INPUT$(1)
    dataf = ASC(aS) - 48
  LOOP UNTIL (dataf = 1 OR dataf = 2)
END SUB

SUB dataselect (j%, xy())
  'Selects 25 equispaced data points and converts coded X,Y to
  'extension ratio, engineering stress and true stress.

  stp% = ndp(j%) \ sndp(j%)
  FOR I% = 1 TO sndp(j%) - 1
    lam(j%, I%) = lambda(xy(1, I%))
    feng(j%, I%) = engstress(j%, xy(2, I%))
    ftrue(j%, I%) = lam(j%, I%) * feng(j%, I%)
  NEXT I%
  lam(j%, sndp(j%)) = lambda(xy(1, ndp(j%)))
  feng(j%, sndp(j%)) = engstress(j%, xy(2, ndp(j%)))
  ftrue(j%, sndp(j%)) = lam(j%, sndp(j%)) * feng(j%, sndp(j%))

  CALL prtselect(j%)
END SUB

SUB datasumm (title$, subtitle$, tdate$)
  'Summarises the input data.

  CLS
  CALL headframe(1, 1, 80, 25)
  CALL centprt(3, " RAW DATA BEFORE AVERAGING.")
  LOCATE 5, 20: PRINT " Base file name: "; filename
  LOCATE 6, 20: PRINT " Title and subtitle: "; title$; SPC(3); subtitle$
  LOCATE 7, 20: PRINT " Test date: "; tdate$
  LOCATE 9, 20: PRINT " Load range = "; loadrng; " N"
  LOCATE 10, 20: PRINT " Extension range = "; extrng; " %"
  LOCATE 11, 20: PRINT " Test speed = "; speed; " mm/min"

  LOCATE 13, 20: PRINT " Number of samples = "; samples
  lin% = 13
  FOR I% = 1 TO samples
    lin% = lin% + 1
    LOCATE lin%, 25: PRINT "Sample no: "; I%; SPC(3); ndp(I%); " data points"
  NEXT I%
  lin% = lin% + 1
LOCATE 24, 59: PRINT "Any key to continue.;
as$ = readkey$
END SUB

FUNCTION datemod$
'Changes the internal date format (mm-dd-yyyy) to (dd mon yy).
'A shared array mon(12)*3 must be initialised in the main program
'with the months "Jan", "Feb" etc.
'See RUPEC.BAS
'
END FUNCTION

FUNCTION datvall (vnum$, iok%)
'Accepts a string representing a number, checks its validity
'and returns the number, with:
'  vnum$ = the string;
'  vnum = the required number;
'  iok% = 0 if the string is invalid, otherwise iok% = 1.
'See RUPEC.BAS
'
END FUNCTION

SUB decide (dec%)
'Allows termination, sample choice and/or polynomial terms selection.
'
CLS
CALL headframe(1, 1, 80, 25)
DO
  CALL headframe(5, 20, 40, 13)
  dec% = 0
  a$ = affirm$(7, 22, "Continue with the analysis")
  IF (a$ = "y") THEN
    a$ = affirm$(9, 22, "Select samples")
    IF (a$ = "y") THEN
      dec% = dec% + 1
      LOCATE 9, 22: PRINT SPACE$(25);
      LOCATE 9, 22: PRINT "Samples to be selected.");
    ELSE
      LOCATE 9, 22: PRINT SPACE$(25);
      LOCATE 9, 22: PRINT "All samples will be included.");
    END IF
    a$ = affirm$(11, 22, "Original Turner function")
    IF (a$ = "y") THEN
      tnr = 1
      LOCATE 11, 22: PRINT SPACE$(34);
      LOCATE 11, 22: PRINT "Original function selected.");
    ELSE
      tnr = 2
      LOCATE 11, 22: PRINT SPACE$(34);
      LOCATE 11, 22: PRINT "Modified function selected.");
    END IF
    IF (tnr = 2) THEN
      a$ = affirm$(13, 22, "Select polynomial terms")
      IF (a$ = "y") THEN
        dec% = dec% + 2
        LOCATE 13, 22: PRINT SPACE$(34);
        LOCATE 13, 22: PRINT "Terms will be selected.");
      ELSE
        LOCATE 13, 22: PRINT SPACE$(34);
        LOCATE 13, 22: PRINT "All terms selected.");
      END IF
      END IF
    ELSE
      dec% = -1
      'terminate
    END IF
  END IF
  a$ = affirm$(15, 22, "Correct")
LOOP UNTIL (a$ = "y")
END SUB
FUNCTION delay! (dt)
'Provides a delay of dt seconds.
'See RUPEC.BAS

END FUNCTION

SUB drawpts (k%, d%, title$)
'Draws a graph of true stress v. lambda, with the experimental
data marked, and the fitted curve if d%=2.

xmax = 0: ymin = 1E+10: ymax = 0
FOR j% = 1 TO samples
  IF (chd(j%) = 1) THEN
    IF (lam(j%, sndp(j%)) > xmax) THEN xmax = lam(j%, sndp(j%))
    SELECT CASE gr
      CASE 1
        yhigh = ftrue(j%, sndp(j%)): ylow = ftrue(j%, 1)
      CASE 2
        yhigh = feng(j%, sndp(j%)): ylow = feng(j%, 1)
      CASE 3
        CALL highlow(j%, yhigh, ylow)
      CASE 4
        CALL highlow(j%, yhigh, ylow)
    END SELECT
    IF (yhigh > ymax) THEN ymax = yhigh
    IF (ylow < ymin) THEN ymin = ylow
  END IF
NEXT j%
IF (19) THEN
  ymax = alog!(ymax): ymin = alog!(ymin)
ELSE
  ymin = 0
END IF
lammax = xmax: 'maximum lambda
IF (k% = 1) THEN
  VIEW
  SCREEN 12
  CLS
  PAINT (100, 100), 1
  LINE (7, 5)-(633, 475), 14, B
  COLOR 14
  ELSE
    VIEW
    CLS
    PAINT (100, 100), 0
    LINE (15, 10)-(625, 470), 15, B
    COLOR 15
  END IF
CALL graphtitle
LOCATE 5, 5: 'compound name
PRINT "Compound: "; title$
LOCATE 7, 14: 'ordinate label
IF (gr < 3) THEN
  PRINT "Stress"
ELSE
  PRINT "Modulus"
END IF

yymin = 0: yymax = ymax
IF (lg) THEN
  yymax = 10 ^ yymax: 'antilog
  yymax = 10 ^ yymax: 'antilog
END IF
LOCATE 8, 9
PRINT USING "(#.# to ###.##)"; yymin; yymax;
LOCATE 24, 53
PRINT USING "Lambda (1 to #.#)"; lammax;

PSET (200, 352)
LINE -(560, 352)
PSET (200, 352)
LINE -(200, 100)
\[
x_{sc} = \frac{360}{(\lambda_{max} - 1)} \\
y_{sc} = \frac{252}{(y_{max} - y_{min})}
\]

'lambda scaling factor (x)' 
'stress factor (y)'

VIEW SCREEN (190, 50)-(565, 355) 'limit graphics

IF (d\% = 2) THEN
CALL calcuv(k\%, ymin, x_{sc}, y_{sc})
CALL headframe(26, 52, 21, 3)
LOCATE 27, 54
PRINT USING "RMS = #.####\ldots\ldots\ldots; sd;"
END IF

CALL exppts(d\%, k\%, ymin, x_{sc}, y_{sc}) 'experimental points
CALL legend(d\%, k\%) 'legend

END SUB

FUNCTION drive$ (ln\%, col\%)
' Returns a disc drive letter, A, B, C or D.
'See RUPEC.BAS

FUNCTION engstress! (j\%, yval)
' Converts a coded stress to an engineering stress.
' engstress! = yval * loadrng / (wdth(j\%) * thk(j\%) * 1000)

END FUNCTION

SUB exppts (d\%, k\%, ymin, x_{sc}, y_{sc})
' Graphs the selected experimental lambda-stress data points.

COLOR 14 'yellow
IF (k\% = 2) THEN COLOR 15 'white

FOR j\% = 1 TO samples
  IF (d\% = 1 OR chd(j\%) = 1) THEN
    FOR I\% = 1 TO sndp(j\%) 'data points
      xp% = 200 + CINT((lam(j\%, I\%) - 1) * x_{sc})
      IF (lg) THEN 'log scale
        SELECT CASE gr
          CASE 1 'true stress
            ydiff = alog!(ftrue(j\%, I\%)) - ymin
          CASE 2 'eng.stress
            ydiff = alog!(feng(j\%, I\%)) - ymin
          CASE 3 'true modulus
            ydiff = alog!(ftrue(j\%, I\%) / (lam(j\%, I\%) - 1)) - ymin
          CASE 4 'eng.modulus
            ydiff = alog!(feng(j\%, I\%) / (lam(j\%, I\%) - 1)) - ymin
        END SELECT
      ELSE
        SELECT CASE gr
          CASE 1 'true stress
            ydiff = ftrue(j\%, I\%) - ymin
          CASE 2 'eng.stress
            ydiff = feng(j\%, I\%) - ymin
          CASE 3 'true modulus
            ydiff = ftrue(j\%, I\%) / (lam(j\%, I\%) - 1) - ymin
          CASE 4 'eng.modulus
            ydiff = feng(j\%, I\%) / (lam(j\%, I\%) - 1) - ymin
        END SELECT
      END IF
      yp% = 352 - CINT(ydiff * y_{sc})
      PSET (xp%, yp%)
    CALL mark(j\%, xp%, yp%) 'symbol
    NEXT I\%
  END IF
NEXT j\%
END SUB
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SUB failsave

'Calculates the mean failure conditions and writes them to the ".tnr" file.

fmt1S = "##.###": fmt2S = "##.###": fmt3S = "###.###"

'All samples.

sumlam = 0: sumfeng = 0: sumftrue = 0
FOR j% = 1 TO samples
    sumlam = sumlam + lammx(j%)
    sumfeng = sumfeng + fengmx(j%)
    sumftrue = sumftrue + ftruemx(j%)
NEXT j%

sumlam = sumlam / samples
sumfeng = sumfeng / samples
sumftrue = sumftrue / samples
PRINT #2, "For all samples:"
PRINT #2, TAB(10); "Lambda";
PRINT #2, TAB(30); "Eng.Stress (MPa)"
PRINT #2, TAB(50); "True Stress (MPa)"
FOR j% = 1 TO samples
    PRINT #2, TAB(10); j%; TAB(15);
    PRINT #2, USING fmt2S; lammx(j%);
    PRINT #2, TAB(35); : PRINT #2, USING fmt2S; fengmx(j%);
    PRINT #2, TAB(55); : PRINT #2, USING fmt3S; ftruemx(j%)
NEXT j%
PRINT #2, TAB(8); "Mean";
PRINT #2, USING fmt2S; sumlam;
PRINT #2, TAB(35); : PRINT #2, USING fmt2S; sumfeng;
PRINT #2, TAB(55); : PRINT #2, USING fmt3S; sumftrue

'Selected samples.

sumlam = 0: sumfeng = 0: sumftrue = 0
k% = 0
FOR j% = 1 TO samples
    IF (chd(j%) = 1) THEN
        k% = k% + 1
        sumlam = sumlam + lammx(j%)
        sumfeng = sumfeng + fengmx(j%)
        sumftrue = sumftrue + ftruemx(j%)
    END IF
NEXT j%

sumlam = sumlam / k%
sumfeng = sumfeng / k%
sumftrue = sumftrue / k%

CALL work(sumlam, sed) 'calculate S.E.D.

PRINT #2, "For selected samples:"
PRINT #2, TAB(10); "Lambda";
PRINT #2, TAB(30); "Eng.Stress (MPa)"
PRINT #2, TAB(50); "True Stress (MPa)"
FOR j% = 1 TO samples
    IF (chd(j%) = 1) THEN
        PRINT #2, TAB(10); j%; TAB(15);
        PRINT #2, USING fmt2S; lammx(j%);
        PRINT #2, TAB(35); : PRINT #2, USING fmt2S; fengmx(j%);
        PRINT #2, TAB(55); : PRINT #2, USING fmt3S; ftruemx(j%)
    END IF
NEXT j%
PRINT #2, TAB(8); "Mean";
PRINT #2, USING fmt2S; sumlam;
PRINT #2, TAB(35); : PRINT #2, USING fmt2S; sumfeng;
PRINT #2, TAB(55); : PRINT #2, USING fmt3S; sumftrue
PRINT #2, TAB(8); "S.E.D.";
PRINT #2, USING "##.###^^^^ MJ/m^3"; sed
END SUB

SUB fopen (f%, flnm$)

'Opens a file, initially after requesting the drive
'and directory/sub-directory.
'See RUPEC.BAS

END SUB
SUB forcext (title$)

'Reads and prepares force-extension data, read from a "*.fex" file.

INPUT #1, title$  
INPUT #1, samples  
IF (samples > 9) THEN
CLS
CALL headframe(1, 1, 80, 25)
CALL headframe(5, 23, 34, 4)
CALL cenprt(5, " DATA ERROR ")
LOCATE 6, 25
PRINT USING "Too many samples (##)"; samples
CALL cenprt(7, "Any key to quit.")
a$ = readkey$
COLOR 7, 0
CLS
STOP
END IF

INPUT #1, glength  
FOR j% = 1 TO samples
INPUT #1, wdth(j%), thk(j%)  
INPUT #1, sndp(j%)  
IF (sndp(j%) > 25) THEN
CLS
CALL headframe(1, 1, 80, 25)
CALL headframe(5, 23, 34, 5)
CALL cenprt(5, " DATA ERROR ")
LOCATE 6, 25
PRINT USING "Too many data points (1(/1)"); sndp(j%)
LOCATE 7, 25
PRINT USING "for sample number #."); j%
CALL cenprt(8, "Any key to quit.")
a$ = readkey$
COLOR 7, 0
CLS
STOP
END IF

FOR I% = 1 TO sndp(j%)
INPUT #1, force, extension
lam(j%, I%) = 1 + extension / glength
feng(j%, I%) = force / (wdth(j%) * thk(j%))
ftrue(j%, I%) = feng(j%, I%) * lam(j%, I%)
NEXT I%

lammx(j%) = lam(j%, sndp(j%))  
' save failure point
fengmx(j%) = feng(j%, sndp(j%))
ftruemx(j%) = ftrue(j%, sndp(j%))

NEXT j%
CLOSE #1
END SUB

SUB graphtitle

'Displays the title of the graph type.

LOCATE 3, 30
IF (lg) THEN
SELECT CASE gr
CASE 1
gtitle$ = "LOG(TRUE STRESS) V. LAMBDA"
CASE 2
gtitle$ = "LOG(ENGINEERING STRESS) V. LAMBDA"
CASE 3
gtitle$ = "LOG(TRUE SECANT MODULUS) v. LAMBDA"
CASE 4
gtitle$ = "LOG(ENGINEERING SECANT MODULUS) v. LAMBDA"
END SELECT
ELSE
SELECT CASE gr
CASE 1
gtitle$ = "TRUE STRESS (MPa) V. LAMBDA"
CASE 2
gtitle$ = "ENGINEERING STRESS V. LAMBDA"
CASE 3
gtitle$ = "TRUE SECANT MODULUS V. LAMBDA"
CASE 4
gtitle$ = "ENGINEERING SECANT MODULUS V. LAMBDA"
END SELECT
END IF
END SUB
SUB headframe (tr%, lc%, wd%, dp%) 'Draws a simple box around headings.
'See RUPEC.BAS

SUB highlow (j%, yhigh, ylow)
'Finds the maximum and minimum secant moduli.

yhigh = 0: ylow = 1E+10
IF (gr = 3) THEN
  FOR I% = 1 TO 25
    yy = ftrue(j%, I%) / (lam(j%, I%) - 1)
    IF (yy > yhigh) THEN yhigh = yy
    IF (yy < ylow) THEN ylow = yy
  NEXT I%
ELSEIF (gr = 4) THEN
  FOR I% = 1 TO 25
    yy = feng(j%, I%) / (lam(j%, I%) - 1)
    IF (yy > yhigh) THEN yhigh = yy
    IF (yy < ylow) THEN ylow = yy
  NEXT I%
END IF

FUNCTION lambda! (xval)
'Converts a recorded coded strain to an extension ratio.

lambda! = 1 + xval * extrng / 1000000!

END FUNCTION

SUB legend (d%, k%)
'Creates the legend for the curves.

cl% = 14
IF (k% = 2) THEN cl% = 15
COLOR cl%
VIEW SCREEN (30, 150)-(140, 310), , cl% 'another window
FOR j% = 1 TO samples
  IF (d% = 1 OR chd(j%) = 1) THEN
    LOCATE 10 + j%, 6
    PRINT "Sample"; j%
    xp% = 128
    yp% = 152 + j% * 16
    PSET (xp%, yp%)
    CALL mark(j%, xp%, yp%)
  END IF
NEXT j%

SUB listres (title$)
'Displays the fitted results as a table, every other point.

fmt1$ = "##", fmt2$ = "######", fmt3$ = "######---"
lamsig(1, 0) = 1: lamsig(2, 0) = 0 'origin
CLS
CALL headframe(1, 1, 80, 25)
LOCATE 2, 10
PRINT "Compound: "; title$
LOCATE 4, 20: PRINT "Lambda";
LOCATE 36: PRINT "Eng.Stress (MPa)";
LOCATE 58: PRINT "True Stress (MPa)"
I% = 0 'origin
LOCATE , 10: PRINT USING fmt1$, I%; 'index
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LOCATE , 20: PRINT USING fmt2$: lamsig(1, I%);  'lambda (1)
LOCATE , 39: PRINT USING fmt3$: lamsig(2, I%);  'true stress (0)
LOCATE , 61: PRINT USING fmt3$: lamsig(2, I%) / lamsig(1, I%);  
FOR I% = 1 TO 25 STEP 2
LOCATE , 10: PRINT USING fmt1$: I%;
LOCATE , 20: PRINT USING fmt2$: lamsig(1, I%);  
LOCATE , 39: PRINT USING fmt3$: lamsig(2, I%) / lamsig(1, I%);  
NEXT I%

'Tension polynomial coefficients.

fmt$ = "##.#f#^^^^"
IF (tnr = 1) THEN
 In% = 20: k% = 0: col% = 40
FOR I% = 1 TO 4
 SELECT CASE I%
 CASE 1
txt$ = "Pretension = "
 CASE 2
txt$ = "Linear coefficient = "
 CASE 3
txt$ = "Power coefficient = "
 CASE 4
txt$ = "Power index = "
END SELECT
 k% = k% + 1
 IF (col% = 40) THEN
 ln% = ln% + 1: col% = 4
 ELSE
 col% = 40
 END IF
LOCATE ln%, col%  PRINT txt$:
SELECT CASE I%
 CASE 1
 PRINT USING fmt$: pretension;
 CASE 2
 PRINT USING fmt$: econst1;
 CASE 3
 PRINT USING fmt$: econst2;
 CASE 4
 PRINT USING fmt$: np;
NEXT I%
ELSEIF (tnr = 2) THEN
 ln% = 20
 LOCATE ln%, 4
 PRINT USING "Equation constant = ##.##^^^^"; bb(0)
 k% = 0: col% = 40
 FOR I% = 1 TO 5
 IF (term(I%) = 1) THEN
 SELECT CASE I%
 CASE 1
txt$ = "Linear coeff. = "
 CASE 2
txt$ = "Quadratic coeff. = "
 CASE 3
txt$ = "Cubic coeff. = "
 CASE 4
txt$ = "Quartic coeff. = "
 CASE 5
txt$ = "Quintic coeff. = "
END SELECT
 k% = k% + 1
 IF (col% = 40) THEN
 ln% = ln% + 1: col% = 4
 ELSE
 col% = 40
 END IF
LOCATE ln%, col%
 PRINT txt$: : PRINT USING fmt$: bb(k%);
END IF
NEXT I%
END IF

ln% = ln% + 1
LOCATE ln%, 4
PRINT USING "S.D. of scatter = #.###^^^ (d.of f.=#.###)"; sd; tndp;

LOCATE 24, 59: PRINT "Any key to continue.";
a$ = readkey$
END SUB

SUB mark (j%, xp%, yp%)

'Draws a symbol.

SELECT CASE j%
CASE 1
   DRAW "bu3 r3 d6 16 u6 r3" 'square
CASE 2
   DRAW "be3 g6 be3 bh3 f6" 'Andrew cross
CASE 3
   DRAW "bu3 d6 bu3 b13 r6" 'George cross
CASE 4
   CIRCLE (xp%, yp%), 3 'circle
CASE 5
   DRAW "u3 bd3 f4 bh4 g4" 'upright triangle
CASE 6
   DRAW "d3 bu3 h4 bf4 e4" 'inverted triangle
CASE 7
   DRAW "bu3 f6 g6 h6 e6" 'diamond
CASE 8
   DRAW "bu4 d8 bu4 b14 r8" 'George cross and circle
   CIRCLE (xp%, yp%), 3
CASE 9
   DRAW "be4 g8 be4 bh4 f8" 'Andrew cross and circle
   CIRCLE (xp%, yp%), 3
END SELECT
END SUB

SUB minresult (fi%, fj%, fk%, fl%)

'Dispays the result of the sweep of parameter ranges.

pretension = rr(1) + (fi% - 1) * (rr(2) - rr(1)) / (npr - 1)
econstl = rr(3) + (fj% - 1) * (rr(4) - rr(3)) / (npr - 1)
econst2 = rr(5) + (fk% - 1) * (rr(6) - rr(5)) / (npr - 1)
nnp = rr(7) + (fl% - 1) * (rr(8) - rr(7)) / (npr - 1)
CLS
CALL headframe(1, 1, 80, 25)
CALL headframe(4, 15, 52, 17)
CALL headframe(5, 27, 27, 3)
LOCATE 6, 29
PRINT "BEST FITTING PARAMETERS";
LOCATE 10, 18
PRINT USING "Initial tension...: #.####"; pretension;
PRINT USING " #.#### to #.####"; rr(1); rr(2)
LOCATE 12, 18
PRINT USING "Linear coefficient: #.####"; econstl;
PRINT USING " #.#### to 0.####"; rr(3); rr(4)
LOCATE 14, 18
PRINT USING "Power coefficient.: #.####"; econst2;
PRINT USING " #.#### to 4.0###"; rr(5); rr(6)
LOCATE 16, 18
PRINT USING "Power index......: #.####"; nnp;
PRINT USING " #.#### to #.####"; rr(7); rr(8)
LOCATE 18, 18
PRINT USING "Root mean square..: #.#####"; ad
LOCATE 20, 30
PRINT " Any key to continue ";
a$ = readkey$
FOR I% = 1 TO 25
   lamsig(1, I%) = 1 + I% * (lammax - 1) / 25
   lam = lamsig(1, I%)
   CALL calcs(laml, stress) 'calculate fitted stress
   lamsig(2, I%) = stress
NEXT I%
END SUB
Appendix C

SUB modequation(param%)
  'Displays the Turner function prior to modification.
  CALL headframe(24, 3, 47, 6)
  COLOR 15
  LOCATE 25, 5: PRINT "T = "
  IF (param% = 0) THEN
    "a" chosen - light red
    COLOR 12
  ELSE
    COLOR 14
  END IF
  PRINT "a";
  k% = 0
  FOR I% = 1 TO 5
    IF (term(I%) = 1) THEN
      k% = k% + 1
      COLOR 15: PRINT "+";
      IF (param% = I%) THEN
        COLOR 12
      ELSE
        COLOR 14
      END IF
      PRINT coeff(I%);
      COLOR 15: PRINT ".e";
      IF (I% > 1) THEN
        1$ = LTRIMS(STR$(1%))
        PRINT "; IS;";
      END IF
    END IF
  NEXT I%
END SUB

SUB modified(title$, dec%, e$)
  'Controls the fit of the modified Turner function.
  IF (dec% = 2 OR dec% = 3) THEN
    CALL terms
  ELSE
    FOR I% = 0 TO 5
      term(I%) = 1
    NEXT I%
  END IF
  FOR j% = 1 TO samples
    FOR I% = 1 TO sndp(j%)
      str(j%, I%) = strain!(j%, I%)
      cosa(j%, I%) = cosine!(j%, I%)
    NEXT I%
  NEXT j%
  nt% = 0
  FOR I% = 1 TO 5
    IF (term(I%) = 1) THEN nt% = nt% + 1
  NEXT I%
  CALL create(nt%)
  nt% = nt% + 1
  CALL assem(nt%)
  nt% = nt% - 1
  CALL solv(nt%)
  CALL control(nt%, title$, e$)
  SCREEN 0
  COLOR 7, 1
END SUB

FUNCTION ok$
  'Returns a 'y' or 'n' answer. Other, invalid answers must be trapped
  'in the calling subroutine.
  'See RUPEC.BAS
END FUNCTION

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Appendix C

'sub origequation (param%)
'displays the Turner function prior to modification.

CALL headframe(24, 3, 40, 6)
COLOR 7
LOCATE 25, 6: PRINT "T = ";
COLOR 14: PRINT "A";
COLOR 7: PRINT " + ";
COLOR 14: PRINT "B";
COLOR 7: PRINT ".e + ";
COLOR 14: PRINT ".e^";
COLOR 14: PRINT "n"
IF (param% = 0) THEN EXIT SUB
COLOR 12
SELECT CASE param%
  CASE 1
    LOCATE 25, 10: PRINT "A"
  CASE 2
    LOCATE 25, 14: PRINT "S"
  CASE 3
    LOCATE 25, 20: PRINT "C"
  CASE 4
    LOCATE 25, 24: PRINT "n"
END SELECT
COLOR 14
END SUB

SUB original (title$, a$)
'controls the fit of the original Turner function.
CALL parameters
IF (ms = 2) THEN
  CALL sweep(fi%, fj%, fk%, fl%)	 'consider the ranges
  CALL minresult(fi%, fj%, fk%, fl%)	 'for parameters
END IF
CALL control(nt%, title$, a$)
'graphing and adjustments
SCREEN 0
're-establish text screen
COLOR 7, 1
END SUB

SUB parameters
'enters the parameters of the original Turner function, either
'as single values or ranges.
CLS
CALL headframe(1, 1, 80, 25)
DO
  CALL headframe(5, 27, 26, 7)
  CALL cenprt(5, " PARAMETERS ")
  CALL headframe(5, 20, 40, 7)
  CALL cenprt(5, " VALUES ")
  FOR I% = 1 TO 4
    loc$ = 5 + I%
    SELECT CASE I%
      CASE 1
        txt$ = " Pretension: "
    CASE 2
        ...
    CASE 3
        ...
    CASE 4
        ...
    END SELECT
    IF (ms = 1) THEN
      CALL headframe(1, 1, 80, 25)
      DO
        CALL headframe(5, 20, 40, 7)
        CALL cenprt(5, " VALUES ")
        FOR I% = 1 TO 4
          i$ = 5 + I%
          SELECT CASE I%
            CASE 1
              txt$ = " Pretension: 
        CASE 2
            ...
        CASE 3
            ...
        CASE 4
            ...
        END SELECT
      END DO
CASE 2
  txt$ = "Linear coefficient: ">
CASE 3
  txt$ = "Power coefficient: ">
CASE 4
  txt$ = "Power index: ">
END SELECT
DO
  LOCATE ln%, 22: PRINT SPACE$(26)
  INPUT "", param$
  param = datval!(param$, iok%)
LOOP UNTIL (iok% = 1)
SELECT CASE I%
  CASE 1
    pretension = param
  CASE 2
    econstl = param
  CASE 3
    econst2 = param
  CASE 4
    nnp = param
END SELECT
NEXT I%
a$ = affirm$(10, 22, "Correct")
LOOP UNTIL (a$ = "Y")
ELSEIF (ms = 2) THEN
  CALL ranges
END IF
END SUB

SUB prtselect (j%)
' Displays selected data as extension ratios and true stresses.

  fmt1$ = "##": fmt2$ = "#.####": fmt3$ = "#.##(#^^^"
CLS
  CALL headframe(1, 1, 80, 25)
  txt$ = "SELECTED DATA - SAMPLE No:" + STR$0%)
  CALL cenprt(3, txt$)
  LOCATE 6, 25: PRINT "Lambda";
  LOCATE , 38: PRINT "Eng.Stress (MPa)"
  FOR I% = 1 TO sndp(j%) STEP 2
    LOCATE , 10: PRINT USING fmt1$; I%;
    LOCATE , 25: PRINT USING fmt2$; lam(j%, I%);
    LOCATE , 41: PRINT USING fmt3$: ftrue(j%, I%)
  NEXT I%
  IF ((sndp(j%) \ 2) * 2 = sndp(j%)) THEN 'an even number
    I% = sndp(j%) - 1
    LOCATE , 10: PRINT USING fmt1$; I%;
    LOCATE , 25: PRINT USING fmt2$; lam(j%, I%);
    LOCATE , 41: PRINT USING fmt3$: ftrue(j%, I%)
  END IF
  CALL check(j%)	 'check the data
  LOCATE 24, 59: PRINT "Any key to continue.";
  a$ = readkey$
  ' save 25 pts to temp data file
  PRINT #3, USING fmt1$; j%
  FOR I% = 1 TO sndp(j%)
    PRINT #3, USING "#.###,(1.11111^^^^; lam(j%, I%); ftrue(j%, I%)
  NEXT
END SUB

SUB ranges
' Enters the limits of the parameters' ranges.

  CALL headframe(1, 1, 80, 25)
  DO
    CALL headframe(5, 15, 50, 18)
    CALL cenprt(5, " PARAMETER RANGES ")
  ...
\[\ln \% = 4\]

```pascal
FOR I\% = 1 TO 9
    \ln \% = \ln \% + 1
    IF ((I\% \div 2) * 2 <> I\%) THEN \ln \% = \ln \% + 1
    LOCATE \ln \%, 17
    PRINT SPACES$(40)
NEXT I\%
```

```pascal
ln\% = 4
FOR I\% = 1 TO 9
    ln\% = ln\% + 1
    IF ((I\% \div 2) * 2 <> I\%) THEN ln\% = ln\% + 1
    LOCATE ln\%, 17
    PRINT SPACES$(40)
NEXT I\%
```

```pascal
SELECT CASE I\%
CASE 1
    prompt$ = "Lower value of the pretension : ")
CASE 2
    prompt$ = "Upper value of the pretension :")
CASE 3
    prompt$ = "Lower value of the linear coeff: ")
CASE 4
    prompt$ = "Upper value of the linear coeff: ")
CASE 5
    prompt$ = "Lower value of the power coeff : ")
CASE 6
    prompt$ = "Upper value of the power coeff : ")
CASE 7
    prompt$ = "Lower value of the power index : ")
CASE 8
    prompt$ = "Upper value of the power index : ")
CASE 9
    prompt$ = "Number per range (max.of 9) : ")
END SELECT
```

```pascal
DO
    LOCATE ln\%, 17
    PRINT SPACES$(40)
    LOCATE ln\%, 17
    PRINT prompt$;
    INPUT "", lim$
    lim = datval!(lim$, iok\%)
    IF lim = 0 THEN lim = .0000001
    LOOP UNTIL (iok\% = 1)
    IF (I\% < 9) THEN
        rr(I\%) = lim
    ELSE
        npr = CINT/hr(I\%)
    END IF
    NEXT I\%
```

```pascal
DO
    LOCATE ln\%, 17
    PRINT SPACES$(40)
    LOCATE ln\%, 17
    PRINT prompt$;
    INPUT "", lim$
    lim = datval!(lim$, iok\%)
    IF lim = 0 THEN lim = .0000001
    LOOP UNTIL (iok\% = 1)
    IF (I\% < 9) THEN
        rr(I\%) = lim
    ELSE
        npr = CINT/hr(I\%)
    END IF
    NEXT I\%
```

```pascal
END IF
NEXT I\%
```

```pascal
a$ = affirm$(20, 17, "Correct")
LOOP UNTIL (a$ = "y")
```

```pascal
logsq = false
a$ = affirm$(21, 17, "Use log(mean square) ")
IF (a$ = "y") THEN logsq = true
END SUB
```

```pascal
SUB readdata
'Dimension xy(2, 500)
FOR I\% = 1 TO 20
    LINE INPUT #1, line$
NEXT I\%
OPEN "C:\temp\tnchoun.csv" FOR OUTPUT AS 113
FOR j\% = 1 TO samples
    FOR k\% = 1 TO 2
        FOR I\% = 1 TO 500
            xy(k\%, I\%) = 0
        NEXT I\%
    NEXT k\%
    LINE INPUT 41, line$
    'skip unwanted lines
    'open temporary file
    'zero temporary array
    'sample heading
    'initialise group count
```

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REd and group data, sacrificing the last point.

FOR I% = 1 TO ndp(j%)
    INPUT #1, xx, yy
    IF (I% = 1) THEN
        xy(1, n%) = xx: xy(2, n%) = yy
        xlast = xx: ylast = yy
        k% = 1
    ELSEIF (I% < ndp(j%)) THEN
        IF (xx <> xlast) THEN
            IF (yy <> ylast) THEN
                xy(1, n%) = xy(1, n%) / k%
                xy(2, n%) = xy(2, n%) / k%
                n% = n% + 1
            END IF
        END IF
        IF (n% > 500) THEN
            CLS
            txt$ = "Sample" + STR$(j%) + " has more then 500 data groups."
            CALL cenprt(5, txt$)
            CALL cenprt(10, "PROGRAM ABANDONED!")
            END
        END IF
        xy(1, n%) = xx: xy(2, n%) = yy
        xlast = xx: ylast = yy
        k% = 1
    ELSEIF (blip% = 1) THEN blip% = 0
    ELSE
        IF (blip% = 0) THEN
            xy(1, n%) = xy(1, n%) + xx
            xy(2, n%) = xy(2, n%) + yy
            k% = k% + 1
        ELSE
            xy(1, n%) = xx: xy(2, n%) = yy
        END IF
        END IF
        ELSE
            blip% = 1
        END IF
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        END If
ELSEIF (tnr = 2) THEN
PRINT #2, TAB(10); title$; SPACE$(5); "(Modified Turner function.)"
PRINT #2,
PRINT #2, TAB(15); USING fmt$; econst1
PRINT #2, TAB(15); USING fmt$; econst2
PRINT W2, TAB(15); USING fmt$; nnp
ELSEIF (term(I%) = 1) THEN
  k% = k% + 1
  PRINT #2, TAB(15); USING fmt$; bb(k%)
ELSE
  PRINT #2, TAB(15); " 0.0000"
END IF
NEXT I%
END IF
PRINT #2, "
PRINT #2, "  
PRINT #2, "  
PRINT #2, TAB(10); "Failure Data."
CALL failsave
PRINT #2, "
PRINT #2, "  
PRINT #2, "  
PRINT #2, TAB(10); title$; TAB(50); newdate$
PRINT #2,
PRINT #2, TAB(10); "Fitted Stress-Strain Data."
CALL stressave(newdate$)
PRINT #2,
PRINT #2, "  
PRINT #2, "  
PRINT #2, "  
PRINT #2, "  
CLOSE #2
LOCATE 24, 8: PRINT "Results filed.;"
LOCATE 24, 59: PRINT "Any key to quit.;"
a$ = readkey$
END SUB
SUB scatter (nt%)
'Calculates the standard deviation of the experimental points
'about the fitted engineering stress v. lambda curve.
  tndp = 0: sumsq = 0
  FOR j% = 1 TO samples
    IF (chd(j%) = 1) THEN
      tndp = tndp + sndp(j%)
    END IF
  NEXT j%
  FOR I% = 1 TO sndp(j%)
    laml = lam(j%, I%)
    CALL calcs(laml, stress)
    engstr = stress / laml
    IF (logsq) THEN
      diff = alog!(feng(j%, I%)) - alog!(engstr)
    ELSE
      diff = feng(j%, I%) - engstr
    END IF
    sumsq = sumsq + diff * diff
  NEXT I%
  sd = SQR(sumsq / tndp)
END SUB
SUB scrtitle
'The title screen for the program.
SCREEN 7
COLOR 15, 1
CALL headframe(2, 2, 38, 23)
LOCATE 4, 4
PRINT "RuPEC";
LOCATE 5, 4
PRINT "Loughborough";
LOCATE 6, 4
PRINT "University";
LOCATE 4, 28
PRINT "RUBBER";
LOCATE 5, 28
PRINT "ELASTICITY";

' t15 = "ANALYSIS OF"
t25 = "UNIAXIAL"
t35 = "STRENGTH TEST"
LOCATE 12, 8
PRINT t15;
LOCATE 14, 8
PRINT t25;
LOCATE 16, 8
PRINT t35;

LOCATE 22, 4
PRINT "JFH/PSO";
dt = delay! (2)
END SUB

SUB solv (nt%)
'Solves assembled equations by triangulation and back substitution.

FOR I% = 0 TO 5
bb(I%) = 0
NEXT I%

FOR k% = 1 TO nt% - 1
I% = k%
FOR I% = k% + 1 TO nt%
IF (ABS(a(I%, k%)) <= ABS(a(1%, k%))) THEN EXIT FOR
I% = I% + 1
NEXT I%
IF (I% <> k%) THEN
FOR j% = k% TO nt%
sw = a(k%, j%)
a(k%, j%) = a(1%, j%)
a(1%, j%) = sw
NEXT j%
sw = c(k%)
c(k%) = c(1%)
c(1%) = sw
END IF
FOR I% = k% + 1 TO nt%
FOR j% = k% + 1 TO nt%
a(I%, j%) = a(I%, j%) - a(k%, j%) * a(I%, k%) / a(k%, k%)
NEXT j%
c(I%) = c(I%) - c(k%) * a(I%, k%) / a(k%, k%)
NEXT I%
NEXT k%

bb(nt%) = c(nt%) / a(nt%, nt%)
FOR I% = nt% - 1 TO 1 STEP -1
ss = 0
FOR j% = I% + 1 TO nt%
ss = ss + a(I%, j%) * bb(j%)
NEXT j%
bb(I%) = (c(I%) - ss) / a(I%, I%)
NEXT I%

'Equation constant.
bb(0) = sum(nt% + 1) / tndp
FOR I% = 1 TO nt%
bb(0) = bb(0) - bb(I%) * sum(I%) / tndp
NEXT I%
END SUB
FUNCTION strain! (j%, I%)
'Calculates the strain in the elastic member for any value of lambda.

\[ e = \sqrt{\frac{(\lambda(j\%, I\%) + 2 / \lambda(j\%, I\%))}{3}} - 1 \]
strain! = e
END FUNCTION

SUB stresssave (newdate$)
'Saves the stress-strain data in the results file.

fmt1$ = "##":  fmt2$ = "#.####":  fmt3$ = "#.^^^^-^^^"
lamsig(1, 0) = 1:  lamsig(2, 0) = 0
PRINT #2, "origin 0"
PRINT #2, TAB(15); "Lambda"; TAB(30); "Eng. Stress (MPa)";
PRINT #2, TAB(50); "True Stress (MPa)"
I% = 0
PRINT #2, TAB(10): PRINT #2, USING fmt1$; I%; 'index
PRINT #2, TAB(15): PRINT #2, USING fmt2$: lamsig(1, I%): 'lambda (1)
PRINT #2, TAB(33): PRINT #2, USING fmt3$: lamsig(2, I%); 'eng. stress (0)
PRINT #2, TAB(53): PRINT #2, USING fmt3$: lamsig(2, I%) / lamsig(1, I%); 'true stress (0)
FOR I% = 1 TO 25
PRINT #2, TAB(10): PRINT #2, USING fmt1$; I%;
PRINT #2, TAB(15): PRINT #2, USING fmt2$: lamsig(1, I%); 'lambda (1)
PRINT #2, TAB(33): PRINT #2, USING fmt3$: lamsig(2, I%) / lamsig(1, I%);
PRINT #2, TAB(53): PRINT #2, USING fmt3$: lamsig(2, I%)
NEXT I%
PRINT #2, ""
'Equation constant = ##.###-^^^"
\[ k = 0 \]
PRINT #2, TAB(10) USING "Equation constant = ##.###-^^^": bb(0)
FOR I% = 1 TO 5
IF (term(I%) = 1) THEN
k% = k% + 1
SELECT CASE I%
CASE 1
txt$ = "Linear coeff. = 
CASE 2
txt$ = "Quadratic coeff. = 
CASE 3
txt$ = "Cubic coeff. = 
CASE 4
txt$ = "Quartic coeff. = 
CASE 5
txt$ = "Quintic coeff. = 
END SELECT
PRINT #2, TAB(10); txt$: : PRINT #2, USING fmt$: bb(k%)
END IF

'Tension polynomial coefficients.
fmt$ = "##.###-^^^"
IF (tnr = 1) THEN
FOR I% = 1 TO 4
SELECT CASE I%
CASE 1
txt$ = "Pretension = 
CASE 2
txt$ = "Linear coefficient = 
CASE 3
txt$ = "Power coefficient = 
CASE 4
txt$ = "Power index = 
END SELECT
PRINT #2, TAB(10): txt$: ; PRINT #2, USING fmt$: pretension
PRINT #2, TAB(33): PRINT #2, USING fmt$: econst1
PRINT #2, TAB(33): PRINT #2, USING fmt$: econst2
PRINT #2, TAB(33): PRINT #2, USING fmt$: nnp
END SELECT
NEXT I%
ELSEIF (tnr = 2) THEN
k% = 0
PRINT #2, TAB(10) USING "Equation constant = ##.###-^^^": bb(0)
FOR I% = 1 TO 5
IF (term(I%) = 1) THEN
k% = k% + 1
SELECT CASE I%
CASE 1
txt$ = "Linear coeff. = 
CASE 2
txt$ = "Quadratic coeff. = 
CASE 3
txt$ = "Cubic coeff. = 
CASE 4
txt$ = "Quartic coeff. = 
CASE 5
txt$ = "Quintic coeff. = 
END SELECT
PRINT #2, TAB(10); txt$: ; PRINT #2, USING fmt$: bb(k%)
END IF
SUB sweep(f1%, f2%, f3%, f4%)

'Five values of each of the four Turner parameters are considered in combination and the minimum mean square of the fit to the experimental data sought.

CLS
CALL headframe(1, 1, 80, 25)
CALL headframe(17, 26, 30, 5)
fi% = 1: f2% = 1: f3% = 1: f4% = 1
sd = 0

minsum = 10000 'a ridiculous minimum SD
FOR ii% = 1 TO npr
pretension = rr(1) + (ii% - 1) * (rr(2) - rr(1)) / (npr - 1)
FOR jj% = 1 TO npr
econst1 = rr(3) + (jj% - 1) * (rr(4) - rr(3)) / (npr - 1)
FOR kk% = 1 TO npr
 . econst2 = rr(5) + (kk% - 1) * (rr(6) - rr(5)) / (npr - 1)
FOR ll% = 1 TO npr
    nnp = rr(7) + (ll% - 1) * (rr(8) - rr(7)) / (npr - 1)
    FOR I% = 1 TO 25
        lamsig(1, I%) = 1 + I% * (lammax - 1) / 25
        laml = lamsig(1, I%)
        CALL calcs(laml, stress) 'calculate fitted stress
        lamsig(2, I%) = stress
    NEXT I%
    CALL scatter(nt%) 'wrt eng.stress
    'Display progress.
    LOCATE 18, 31
    PRINT USING "# 	 "; ii%; jj%; kk%; ll%;
    IF (sd < minsum) THEN
        minsum = sd
        fi% = i1%; f2% = j1%; f3% = k1%; f4% = l1% 'note
    END IF
   LOCATE 19, 31
    PRINT USING "# 	 "; i1%; j1%; k1%; l1%;
    LOCATE 20, 28
    PRINT USING "Mean square = #.####-"; sd
    END IF
NEXT ll%
NEXT kk%
NEXT jj%
NEXT i1%
END SUB

SUB terms
'The terms, to a maximum of a quintic are chosen.

DO
CLS
CALL headframe(1, 1, 80, 25)
term(0) = 1
FOR I% = 1 TO 5
    term(I%) = 0
NEXT I%
CALL headframe(1, 1, 80, 25)
term(0) = 1
FOR I% = 1 TO 5
    term(I%) = 0
NEXT I%
CALL cenprt(5, " POLYNOMIAL TERMS ")
as$ = affirm$(6, 25, " Include linear term")
IF (as$ = "y") THEN term(1) = 1
as$ = affirm$(7, 25, "Include quadratic term")
IF (as$ = "y") THEN term(2) = 1
as$ = affirm$(8, 25, " Include cubic term")
IF (as$ = "y") THEN term(3) = 1
as$ = affirm$(9, 25, " Include quartic term")
IF (as$ = "y") THEN term(4) = 1
as$ = affirm$(10, 25, " Include quintic term")

NEXT I%
END SUB
IF (a$ = "y") THEN term(5) = 1
a$ = affirm$(11, 25, "Correct")
LOOP UNTIL (a$ = "y")
END SUB

SUB widththk (j%)
'Enters the width and thickness of a sample, the former defaulting to 4 mm unless rejected.
CLS
CALL headframe(1, 1, 80, 25)
CALL headframe(6, 20, 40, 11)
txt$ = "Sample No:" + ST$(j%)
CALL cenprt(8, txt$)
LOCATE 10, 25: PRINT "Enter:"
try% = 0
DO
  FOR ln% = 11 TO 14
    LOCATE ln%, 22: PRINT SPACES(36)
  NEXT ln%
  IF (try% = 0) THEN 'default width
    LOCATE 11, 30: PRINT "Sample width (mm): 4"
    wdth(j%) = 4
  ELSE
    DO
      LOCATE 11, 30: PRINT SPACE$(25)
      LOCATE 11, 30: INPUT "Sample width (mm): ", wd$,
      thk(j%) = datval!(wd$, iok%)
    LOOP UNTIL (iok% = 1)
  END IF
  DO
    LOCATE 12, 30: PRINT SPACES(25)
    LOCATE 12, 30: INPUT "Thickness (mm): ", thk$
    LOOP UNTIL (iok% = 1)
    a$ = affirm$(14, 25, "Correct")
    IF (a$ = "n") THEN try% = try% + 1
    LOOP UNTIL (a$ = "y")
  END SUB

SUB work (sumlam, sed)
'Determines the strain energy density for the averaged failure point.
A cubic metre of the compound is considered before distortion.

DIM force(24)

dlam = (sumlam - 1) / 24 'lambda or extension increment
FOR I% = 1 TO 24
  laml = 1 + I% * dlam 'lambdas
  CALL calcs(laml, stress)
  force(I%) = stress / laml 'eng.stress or force
NEXT I%

integ = 0 'Simpson's rule
FOR I% = 1 TO 23
  IF ((I% \ 2) * 2 <> I%) THEN
    integ = integ + 4 * force(I%)
  ELSE
    integ = integ + 2 * force(I%)
  END IF
NEXT I%
integ = integ + force(24)
sed = dlam * integ / 3 'workdone or S.E.D.
Appendix D
BXLAS.BAS

Laser Biaxial Test Apparatus Control Software.
(c) June to Sept 1995, Jan 1996
Revision 6
J Hallett, P S Oubridge

Modules Required;
Requires linking with c30M_1.qib
Requires include files pc30.inc, RuPEC.inc, and ErrorMsg.INC
ALSO REQUIRES RUPEC.BAS MODULE

Software reads 3 channels in burst mode at a variable frequency.
First a zero reading is taken, then data is grabbed until failure detected
The <Esc> key ends sampling. The results are written to disk, and plotted
on screen.
Non dma version.
Ongoing zero reading
Converts readings to kPa and mm.
prints values at break

DECLARE SUB Channels ()
DECLARE SUB CompoundName (compound$)
DECLARE SUB cenprt (ln%, btitle$)
DECLARE SUB delay (dt)
DECLARE SUB FileOpen (ext$, flnm$)
DECLARE SUB GetReadings (freq)
DECLARE SUB GetSample (k%, freq!)
DECLARE SUB GetSamples (k%, freq, sampchan%)
DECLARE SUB headframe (tr%, lc%, wd%, dp%)
DECLARE SUB message (1%, mtxt$())
DECLARE SUB minmax (k%, xmin, xmax, ymin, ymax)
DECLARE SUB openscr (txt$(), name$)
DECLARE SUB PlotResults (k%)
DECLARE SUB progress (pnum%)
DECLARE SUB ReviseZero (f%, freg!)
DECLARE SUB SetUpScreen ()
DECLARE SUB ZeroReading (freq)
DECLARE SUB ZeroResults (ok%)

DECLARE FUNCTION affirm$ (ln%, col%, text$)
DECLARE FUNCTION datemod$ ()
DECLARE FUNCTION readkey$ ()

DIM SHARED mon(12) AS STRING * 3
DIM SHARED chanlist(16) AS INTEGER
DIM SHARED result(16), sd(16) AS SINGLE
DIM SHARED sampchan(2)
DIM SHARED time(1 TO 500) AS SINGLE
DIM SHARED press(1 TO 500) AS SINGLE
DIM SHARED height(1 TO 500) AS SINGLE
DIM SHARED mtxs(2)
DIM titles(4)

COMMON SHARED numrd AS INTEGER
COMMON SHARED zettim, zettpr, zettmm AS SINGLE 'in V
COMMON SHARED zecoppa, zeropress, zeromheight AS SINGLE 'in kPa and mm
COMMON SHARED ck, pr AS INTEGER
COMMON SHARED numchan AS INTEGER
Appendix D

CONST z% = 1, r% = 2 'constants for get samples

REM $INCLUDE: 'pc30.inc'
REM $INCLUDE: 'RuPEC.inc'
REM $DYNAMIC

months: DATA "Jan", "Feb", "Mar", "Apr", "May", "Jun"
DATA "Jul", "Aug", "Sep", "Oct", "Nov", "Dec"

RESTORE months
FOR i% = 1 TO 12
READ mon(i%)
NEXT i%

ukdate$ = datemod$

'Opening titles.

title$(1) = "Biaxial"
title$(2) = "Strength"
title$(3) = "Test Data"
title$(4) = "Acquisition."
name$ = "JFH/PSO"

CALL openscr(title$(1), name$)
COLOR 7, 1

'Initialise AtoD.

set.base (&H700)
CLS
CALL headframe(1, 1, 80, 25)

IF diag% = ok.30 THEN
mtxt$(1) = "PC-30 Board OK"
CALL message(1, mtxt$(1))
ELSE
mtxt$(1) = "Initialisation of PC-30 Failed"
mtxt$(2) = "Please check hardware"
CALL message(2, mtxt$(1))
END IF
CALL delay(2)

'Define channels of data logger and set clock.

CALL Channels
freq = 10
CALL SetClock(freq)

'Compound Loop

DO 'loop while there is another compound

CALL CompoundName(compound$)
PRINT 1(1, "Biaxial Test Results File for " + compounds
PRINT #1, "Test Date: " + ukdate$

'test date

'Compound Loop.

testno% = 0
DO ' while there is another sample
  testno% = testno% + 1
  IF testno% > 1 THEN PRINT #1, "-1, -1"
  PRINT #1, USING "Run Number: #"; testno%
  PRINT #1, "Time,Height,Pres,Inten"
  PRINT #1, "(s), (mm), (kPa), (\%), (\%)"
\texttt{Zero readings.}

\texttt{Get readings.}

\texttt{Display graph of readings.}

\texttt{Check for another test.}

\texttt{Reset screen.}

---SUBROUTINES---

\texttt{REM $STATIC}

\texttt{SUB Channels}

\texttt{DO}

\texttt{SUB CompoundName (compound$)}

\texttt{END SUB}

\texttt{END}
SUB Get1Sample (r%, freq)
',
'Subroutine to take 1 sample from the required channels
'in burst mode to normal memory.
'Frequency and number of samples per channel given.
',
REM $DYNAMIC

DIM reading$(16)
DIM mtxt$(2)
',

************
'Initialise.
************

ad.prescaler (pre)  'clock prescaler
ad.clock (clk)      'sets clock to reqd val

************
'Get data.
************
samptime(1) = TIMER   'start time
iret% = mb.chan(chanlist(1), numchan, numchan, reading$(1))

samptime(2) = TIMER   'end time
IF iret% <> 0 THEN
  mtxt$(1) = "An error has occured"
  CALL message(1, mtxt$())
END IF

END SUB
REM $STATIC
SUB GetReadings (freq),
'Subroutine to get real sample data until pressure hits zero
REM $DYNAMIC

DIM mtxt$(2)

*************
'Initialisation.
*************

LOCATE 6, 34; PRINT "Getting Data"

flag% = 0  'set burst flag
flagl% = 0  'set test start flag
press2 = 0  'zero pressure
sampperchan% = 10  'set samples to be averaged

*************
'Start sampling data.
*************

DO
  press1 = press2
  CALL Get1Sample(r%, freq)  'get data
  IF result(3) < 3 THEN
    LOCATE 8, 33: COLOR 20, 1
    PRINT "Intensity Low"
    COLOR 7, 1
  ELSE
    LOCATE 8, 33: PRINT SPACES$(13)
  END IF

END IF

******************************************************************************
'Test started so calc and print values.
******************************************************************************
numrd = numrd + 1 "increment counter
result(3) = result(3) * 100 / 4.5 "convert intensity into %

'Averagetimeofsample.
'
time(numrd) = ((samptime(2) - samptime(1)) / 2) + samptime(1)
time(numrd) = time(numrd) - starttime "time relativeto start
'of test

'convert pressure and height into "real" units.

press(numrd) = (result(2) * 101.375) - zeroppa "convert P to kPa
height(numrd) = (30 * ABS(result(1) - 5)) - zerohtmm "convert height to mm

PRINT #1, USING "0110.##,(1.##,11##.(1,###.##,); time(numrd); result(1); height(numrd); 0;
PRINT 1(1, USING "0.11#,###.(1(1,###.##,ON.#"; result(2); press(numrd); 0; result(3)

'Progress indicator andbreak detection.

IF ((numrd \ 4) * 4 = numrd) THEN "every fourth reading
pnum% = numrd / 4
CALL progress(pnum%)
END IF

press2 = result(2)
IF (press2 < (zeropress + .05) AND pressl > (zeropress + .1)) THEN 'check for
burst
flag% = 1

Sample limit and Esc key detection.

ELSEIF numrd = 500 THEN
flag% = 1
mtxt$(1) = "Sample limit reached"
mtxt$(2) = "Press any key to continue"
CALL message(2, mtxt$())
dummy$ = readkey$
END IF

IF INKEY$ = CHR$(27) THEN flag% = 1
LOOP WHILE flag% <> 1
END SUB

REM $STATIC
SUB GetSamples (k%, freq, chansamp%) 

'Subroutine to take and average samples from the required channels
'in burst mode to normal memory.
'Frequency and number of samples per channel given.
'500 samples maximum.
'
REM $DYNAMIC

DIM reading%(500)
DIM mtxt$(2)
DIM x(0 TO 16) AS LONG
DIM xx(0 TO 16) AS LONG
DIM minread(0 TO 16) AS INTEGER
DIM maxread(0 TO 16) AS INTEGER

nsamp% = numchan * chansamp% "total number of samples

**********
'Initialise.

**********

IF (nsamp% > 500) THEN 'check number of samples
mtxt$(1) = "Too many samples" 'fits in memory
mtxt$(2) = "Required"
CALL message(2, mtxt$())
END IF

ad.prescaler (pre)
ad.clock (clk) 'clock prescaler
'sets clock to reqd val
Appendix E

'Get data.

samptime(1) = TIMER 'start time
iret% = mb.chan(chanlist(1), nsamp%, numchan, reading%(1))
samptime(2) = TIMER 'end time
IF iret% <> 0 THEN
    mtxt$(1) = "An error has occured"
    CALL message(1, mtxt$(1))
END IF

'Average data.

FOR j% = 0 TO numchan - 1
    x(j%) = 0: xx(j%) = 0
    maxread(j%) = 0
    minread(j%) = 4096
NEXT j%

FOR i% = 1 TO nsamp% STEP numchan 'sum data
    FOR j% = 0 TO numchan - 1
        x(j%) = x(j%) + reading%(i% + j%) 'normal
        IF (k% = 1) THEN 'zeroing run
            xx(j%) = xx(j%) + reading%(i% + j%) ^ 2 'squared
        ELSE 'testing run
            IF reading%(i% + j%) > maxread(j% + 1) THEN 
                maxread(j% + 1) = reading%(i% + j%) 'new maximum
            ELSEIF reading%(i% + j%) < minread(j% + 1) THEN
                minread(j% + 1) = reading%(i% + j%) 'new minimum
        END IF
    NEXT j%
NEXT i%

FOR i% = 1 TO numchan
    result(i%) = x(i% - 1) / chansamp% 'average
    IF (k% = 1) THEN 'zero S.D.
        sd(i%) = xx(i% - 1) - (x(i% - 1) ^ 2 / chansamp%) 
        sd(i%) = SQR(sd(i%) / (chansamp% - 2 - chansamp%))
    ELSE
        sd(i%) = ((maxread(i%) - minread(i%)) / result(i%)) * 100
    END IF
NEXT i%

'Convert data to volts.

FOR i% = 1 TO numchan
    result(i%) = ((result(i%) AND 61-IFFF) - 2048) * 5 / 2048
    sd(i%) = sd(i%) * 10 / 4096
NEXT i%

END SUB

SUB minmax (k%, xmin, xmax, ymin, ymax)

'Find the minimum and maximum values to be plotted.

    xmin = 0: xmax = -1E+10 'ridiculous maxima and zero minima
    ymin = 0: ymax = -1E+10

FOR i% = 1 TO numrd
    IF (time(i%) > xmax) THEN 
        xmax = time(i%)
    END IF
    IF (k% = 1) THEN
        IF (ABS(height(i%) - 5) > ymax) THEN
            ymax = ABS(height(i%) - 5)
        ELSEIF press(i%) > ymax THEN
            ymax = press(i%) 
        END IF
    END IF
NEXT i%

END SUB
SUB PlotResults (k%)

; Subroutine to plot logged data.

; Plot zero.

maxheight = 0
maxpres% = 0
maxnum% = 0

IF (k% = 1) THEN
pt2 = height(1)
ELSE
pt2 = press(1)
END IF
LINE (0, 0)-(time(1), pt2)

; Plot results

FOR i% = 1 TO numrd - 1
    IF (k% = 1) THEN
        pt1 = height(i%)
        pt2 = height(i% + 1)
        IF pt2 > maxheight THEN
            maxheight = pt2
            maxnum% = i% + 1
        END IF
    ELSE
        pt1 = press(i%)
        pt2 = press(i% + 1)
        IF pt2 > maxpres% THEN
            maxpres% = pt2
            maxnum% = i% + 1
        END IF
    END IF
    LINE (time(i%), pt1)-(time(i% + 1), pt2)
NEXT i%

; Plot burst.

IF (k% = 1) THEN
ptl = maxheight
ELSE
ptl = press(maxnum%)
END IF
LINE (time(numrd), ptl)-(time(numrd), 0), , , &HUH
END

REM $STATIC
SUB progress (pnum%)

; Displays a progress indicator when reading data during a run
; prints values of H and P

ln% = 23 + (pnum% - 1) \ 60
rm% = pnum% MOD 60 + 1
LOCATE ln%, rm%: PRINT CHR$(177)
LOCATE 7, 33: PRINT USING "#.111": result(1)
LOCATE 7, 42: PRINT USING "(MIA": press(numrd); , , &H1111
END

SUB ReviseZero (f%, freq)

; This subroutine constantly updates the zero reading
; until the test starts or esc key pressed (r%=1)

**************
; Define Screen.
**************

CLS
CALL headframe(5, 29, 22, 6)
LOCATE 6, 34: PRINT "Current Zero"
LOCATE 7, 30
PRINT USING "H=1.### V=###.###kPa": zeroheight; zeroppa
LOCATE 9, 30: PRINT "Press Esc to cancel"
Appendix D

Set flags.

---

f% = 0
flag% = 0
press2 = 0
numrd = 0
zerotime = TIMER

---

Start sampling data.

---

DO
sumh = 0: sump = 0: sumi = 0
FOR i% = 1 TO 10
press1 = press2
CALL GetSample(k%, freq)

IF i% = 1 THEN zerostart = samptime(1)

IF result(3) < 3 THEN
LOCATE 8, 33: COLOR 20, 1
PRINT "Intensity Low"
COLOR 7, 1
ELSE
LOCATE 8, 33: PRINT SPACES(13)
END IF

END IF

IF i% = 10 THEN

---

Test started, so write data to disk.

---

END IF

NEXT

Test started so write data to disk.

---

IF f% <> 1 THEN
numrd = 1

---

Average time of sample.

---

time(numrd) = ((samptime(2) - samptime(1)) / 2) + samptime(1)
time(numrd) = time(numrd) - starttime

---
convert pressure and height into "real" units.

\[
\text{press}(\text{numrd}) = (\text{result}(2) \times 101.375) - \text{zeropp} \quad \text{convert P to kPa} \\
\text{height}(\text{numrd}) = (30 \times \text{ABS}([\text{result}(1) - 5]) - \text{zerohtmm} \quad \text{convert height to mm}
\]

PRINT #1, USING "####.##,####.##,#####.###", \text{time}(\text{numrd})\text{;} \text{result}(1); \text{height}(\text{numrd})\text{;} \text{sd}(1)\text{;} \text{result}(2)\text{;} \text{press}(\text{numrd})\text{;} \text{sd}(2)\text{;}

(*)

**********************************************************
' Esc key detection.
**********************************************************

IF INKEY$ = CHR$(27) THEN
\% = 1
\flag$ = 1
END IF
END IF

LOOP WHILE \flag$ <> 1
END SUB

REM SDYNAMIC
SUB SetClock (freq)
' Subroutine to set ad.prescaler and ad.clock to required values
'this MUST be called before sub getsamples and getsample.

'find highest and lowest possible prescale values
\minrem = 1
\scale = 20000000 / freq
'min rem cast
maxpres$ = Fix(\scale / 21)
'highest possible prescale
\text{IF} \maxpres$ > 32767 \text{THEN} \maxpres$ = 32767 \quad \text{32767 largest poss value}
\text{minclock} = \text{CINT}(\scale / 32767)
' lowest possible prescale to give clock < 2^15

'find best clock value
FOR \prescale = \minclock\text{ TO } \maxpres
\clock = \scale / \prescale
'calc clock scale for
\rmd = \clock - \text{Fix}(\clock)
'calc difference between
\text{IF} \rmd < \minrem \text{THEN}
\minrem = \rmd
'min remainder found or clock to slow
\text{clk} = \text{Fix}(\clock)
'set new min remainder
\text{pre} = \text{CINT}(\prescale)
'set prescaler
END IF
\text{IF} \minrem = 0 \text{THEN EXIT FOR}
'best combination found
NEXT

\freq2 = 20000000 / (\text{clk} \times \text{pre})
'!!! \freq2 ??????
END SUB

SUB SetupScreen
'Sets up double window graphics screen

SCREEN 12
VIEW
CLS
\text{PAINT} (100, 100), 1 \quad \text{blue screen}
\text{LINE} (7, 5)-(633, 475), 14, B \quad \text{yellow border}
\text{LINE} (7, 240)-(633, 240), 14 \quad \text{dividing line}

'Graphs of height v.time and pressure v.time.

FOR \k% = 1 TO 2
\text{IF} (\k% = 1) \text{THEN}
\text{VIEW} (40, 20)-(600, 220) \quad \text{graphics window 1}
\text{ELSE}
\text{VIEW} (40, 260)-(600, 460) \quad \text{graphics window 2}
\text{END IF}
\text{COLOR} 14
\text{CALL minimax(k%, x\_{min}, x\_{max}, y\_{min}, y\_{max})} \quad \text{find minimum and maximum}

\text{for X and Y axes}

**********************************************************
'Set screen dimensions and draw axes.
**********************************************************
Appendix D

xwmax = 1.1 * xmax
xwmin = -.1 * xmax
ywmax = 1.1 * ymax
ywmin = -.1 * ymax
WINDOW (xwmin, ywmin)-(xwmax, ywmax)  'use real units

************
'Draw axes.
************

LINE (xmin, 0)-(xmax, 0)  'abscissa
LINE (0, ymin)-(0, ymax)  'ordinate
IF (k% = 1) THEN  'label axes
  LOCATE 12, 64: PRINT "Time";
  LOCATE 3, 14: PRINT "Height";
  LOCATE 4, 11: PRINT USING "max ##.##mm"; ymax
ELSE
  LOCATE 27, 64: PRINT "Time";
  LOCATE 18, 14: PRINT "Pressure";
  LOCATE 19, 12: PRINT USING "max ##.##kPa"; ymax
END IF

CALL PlotResults(k%)

IF (k% = 2) THEN
  LOCATE 30, 58: PRINT "Any key to continue";
  dummy$ = readkey$
END IF
NEXT k%
END SUB

SUB ZeroReading (freq)

'Subroutine to take zero readings.

DIM mtxt$(2)
mtxt$(1) = "Zeroing Equipment"
CALL message(1, mtxt$)
DO 'Loop until zero reading OK, enables simple errors to be corrected
ok% = 0
1

'Get samples.

samplerchan% = 10  '100 readings per channel
CALL GetSamples(z%, freq, samplerchan%)

'Convert readings and print averages and S.Ds.

result(3) = result(3) * 100 / 4.5  'convert intensity into %
sd(3) = sd(3) * 100 / 4.5  'S.D. of intensity
zerotime = 0
zeroheight = result(1)
zerobottom = result(1)
zerohtm = 30 + ABS(result(1) - 5)
zeropress = result(2)
sd(2) = sd(2) * 100 / result(2)  'S.D. of pressure
zeroppa = result(2) * 101.375  'convert pressure to kPa

CALL ZeroResults(ok%)
CALL delay(2)
LOOP UNTIL ok% = 1
PRINT 41, USING "z1,4.44,444.4,444.4,"; zeroheight; 0!; sd(1);
PRINT 41, USING "N.44,444.44,444.4,44.4"; zeropress; 0!; sd(2); result(3)

END SUB

SUB ZeroResults (ok%)

'Displays the results of zeroing.

DIM mtxt$(5)
mtxt$(1) = "Zero Readings Taken (volts)"
mtxt$(2) = "Height =" + STR$(result(1)) + " +/-" + STR$(sd(1)) + "%"
mtxt$(3) = "Pressure =" + STR$(result(2)) + " +/-" + STR$(sd(2)) + "%"
mtxt$(4) = "Intensity =" + STR$(result(3)) + " +/-" + STR$(sd(3)) + "%"

mtxt$(5) = "Any Key to Continue Test"
CALL message(5, mtxt$(1))
a$ = INKEY$

' see if any key pressed

' check results OK
CLS
CALL headframe(13, 27, 21, 3)
a$ = affirm$(14, 28, "Zero OK")
IF a$ <> "" THEN ok$ = 1

' check zero readings OK
' if key pressed then start test

END SUB
Appendix E
SIMDIA.BAS

******************************************************************************
* Program SIMDIA.BAS
* *
* This program accepts the dimensions of a circular rubber diaphragm and the original or modified
* Turner constants of the compound from which it is moulded. For a range of extension ratios at the pole *
* up to and beyond a specified height, the pressures are calculated, together with the strain energy *
* densities at the pole.
* *
*
******************************************************************************

DECLARE SUB box (sd%, tr%, br%, lc%, rc%, btitle$, dr%)
DECLARE SUB calcs (ir%, e%, mtitle$)
DECLARE SUB check (xfnd$)
DECLARE SUB choose (men%, items$, key$, sel%)
DECLARE SUB clamp ()
DECLARE SUB datemod ()
DECLARE SUB delay (dt)
DECLARE SUB diags (d%, m%, j%, kurvc(), xf())
DECLARE SUB dimens ()
DECLARE SUB display (ch%, mtitle$)
DECLARE SUB endrun (e%)
DECLARE SUB excel ()
DECLARE SUB fileconsts (mtitle$, name$)
DECLARE SUB filedat (p%, lame, mtitle$, pstress, energy)
DECLARE SUB fillarray (i%)
DECLARE SUB fopen (f%)
DECLARE SUB focus (xf(), kurvc())
DECLARE SUB interp (xf(), kurvc())
DECLARE SUB lamiter (i%, lamfnd%)
DECLARE SUB lines (f%, nl%)
DECLARE SUB menu (men%, sd%, btitle$, tr%, lc%, sel%)
DECLARE SUB modconfile (name$)
DECLARE SUB modconsts (name$)
DECLARE SUB modcurv (j%, kurvc(), xf{}, cmod1, cmod2, flag%)
DECLARE SUB modF (name$)
DECLARE SUB nextincl (i%, sina, cosa, e%) 
DECLARE SUB origconfile (name$)
DECLARE SUB origconsts (name$)
DECLARE SUB plot (ir%, mtitle$)
DECLARE SUB prnt (lame, mtitle$, pstress, energy)
DECLARE SUB prntout (ir%)
DECLARE SUB prntsum (mtitle$, name$)
DECLARE SUB prof (j%, lame, tenc, xf(), e%)
DECLARE SUB range (ir%)
DECLARE SUB scrtitle ()
DECLARE SUB setmenu (items$())
DECLARE SUB show (d%, od$)
DECLARE SUB specify (mtitle$)
DECLARE SUB stress (lame, pstress)
DECLARE SUB summary (mtitle$, name$)
DECLARE SUB surround (back$)
DECLARE SUB tension ()
DECLARE SUB test (ir%, finish$)
DECLARE SUB volume ()
DECLARE SUB which (nt%)
DECLARE SUB work ()

DECLARE FUNCTION affirm$ (ln%, col$, text$)
DECLARE FUNCTION atitle$ (lnS, coil, OS)
DECLARE FUNCTION datval! (vnum$, iok%)
DECLARE FUNCTION turner! (eztn)
DECLARE FUNCTION ok$ ()
DECLARE FUNCTION readkey$ ()
DIM mon(12) AS STRING * 3
DIM done(12), tn(9) AS INTEGER
DIM tcnst(7), bb(9), apres(17) AS SINGLE
DIM xc(51), yc(51), ang(51) AS SINGLE
DIM curv(2, 51), ten(2, 51), ext(3, 51) AS SINGLE
DIM prevcurv(17) AS SINGLE
DIM xp(17, 51), yp(17, 51), xx(17, 102), yy(17, 102)
DIM htc(18), wd(18), wdh(18), lamax(18), pstr(18), psed(18) AS SINGLE
DIM cccrvr(18), hhfht(18), hhfwd(18), vol(18), twor(18) AS SINGLE

COMMON SHARED mon() AS STRING * 3
COMMON SHARED working AS STRING, basename AS STRING
COMMON SHARED done(), tn() AS INTEGER
COMMON SHARED tcnst(), bb(), apres() AS SINGLE
COMMON SHARED xc(), yc(), ang() AS SINGLE
COMMON SHARED curv(), ten(), ext() AS SINGLE
COMMON SHARED prevcurv() AS SINGLE
COMMON SHARED xp(), yp(), xx(), yy() AS SINGLE
COMMON SHARED htc(), wd(), wdh(), lamax(), pstr(), psed() AS SINGLE
COMMON SHARED cccrvr(), hhfht(), hhfwd(), vol(), twor() AS SINGLE

COMMON SHARED newdate AS STRING * 11
COMMON SHARED count, nterms, nn, ni, mt AS INTEGER
COMMON SHARED kp, profile, disp, pfile AS INTEGER
COMMON SHARED maxht, maxext, rad, thk, delr, lam1, lam2, lam3 AS SINGLE
COMMON SHARED x, y, a, kurv1, kurv2, pres, ten1, ten2 AS SINGLE
COMMON SHARED pi, lengo, xmax, ymax, mg AS SINGLE

REM $DYNAMIC
ON ERROR GOTO message

months: DATA "Jan", "Feb", "Mar", "Apr", "May", "Jun"
DATA "Jul", "Aug", "Sep", "Oct", "Nov", "Dec"
sline: DATA 218, 191, 192, 217, 196, 179
dline: DATA 201, 187, 200, 188, 205, 186
lambda: DATA 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0
DATA 5.5, 6.0, 6.5, 7.0, 7.5, 8.0, 8.5, 9.0, 9.5

RESTORE months
FOR i% = 1 TO 12
READ mon(i%)
'zero all input flags
done(i%) = 0
NEXT i%

'zero all input flags

CALL datemod
'modify the date format
pi = 4 * ATN(1)
'lengo = SQR(3)
'opening diagonal
CALL scrtitle
'restore screen
SCREEN 2
SCREEN 0

CLS
CALL surround(1)
'border

done(1) = 0
FOR i% = 2 TO 10
done(i%) = 1
NEXT i%

'operations loop

DO
CALL menu(1, 2, "MAIN MENU ", 5, 8, sel)

SELECT CASE sel
CASE 1
CALL fopen1()
done(1) = 1: done(2) = 0
CASE 2
count = count + 1
CALL specify(mtitle5)
done(2) = 1: done(3) = 0: done(4) = 0: done(5) = 0:

Page E2
IF count > 1 THEN done(5) = 0 'for 2nd runs on

CASE 3
  'elastic constants
  IF (done(2) = 1) THEN
    CALL origconsts(name$)
    mt = 1 'flag original
    done(3) = 1: done(4) = 1: done(5) = 1: done(6) = 0: done(7) = 0
  END IF

CASE 4
  'elastic constants
  IF (done(2) = 1) THEN
    CALL modconsts(name$)
    mt = 2 'flag modified
    done(3) = 1: done(4) = 1: done(5) = 1: done(6) = 0: done(7) = 0
  END IF

CASE 5
  'modify F
  IF (done(2) = 1) AND (count > 1) THEN
    CALL modF(name$)
    done(3) = 1: done(4) = 1: done(5) = 1: done(6) = 0: done(7) = 0
  END IF

CASE 6
  ir% = 1
  CALL range(ir%) 'height range
  maxext = 9 'maximum lambda
  done(6) = 1: done(7) = 1: done(8) = 0

CASE 7
  ir% = 2
  CALL range(ir%) 'single ext.ratio
  maxht = 1000 'maximum height
  done(6) = 1: done(7) = 1: done(8) = 0

CASE 8
  CALL prntout(ir%) 'output choice
  done(8) = 1: done(9) = 0

CASE 9
  CALL calcs(ir%, e%, mtitle$) 'calculations
  IF (profile > 0) THEN
    IF (pfile = 1) THEN
      CALL fileconsts(mtitle$, name$)
    END IF
  END IF
  END IF
  CALL surround(1)
  done(9) = 1: done(10) = 0

CASE 10
  IF (profile > 0) THEN
    'some profiles OK
    CALL plot(ir%, mtitle$)
    IF (ir% = 1) THEN
      CALL summary(mtitle$, name$)
    END IF
    done(2) = 0 'set input flags
    FOR i% = 3 TO 10
      done(i%) = 1
    NEXT i%
  END IF
  CALL endrun(e%) 'end of run

CASE 11
  SCREEN 0
  COLOR 7, 0
  CLOSE #1, #2, #3, #4
  END
END SELECT
LOOP
END

'Messages of errors not trapped.

message:
COLOR , 11
CALL surround(1)
COLOR 0

LOCATE 8, 30
PRINT "AN ERROR HAS OCCURRED"
LOCATE 13, 10

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PRINT "Note any displayed message:"  
LOCATE 15, 15  
SELECT CASE ERR  
CASE 5  
PRINT "An illegal function call has been made."  
CASE 6  
PRINT "Overflow has occurred,"  
LOCATE 14, 20  
PRINT "- a variable value is too large."  
CASE 9  
PRINT "An array subscript is out of its allowable range."  
CASE 11  
PRINT "An attempt has been made to divide by zero."  
CASE 27  
PRINT "The printer is not on line."  
CASE 52, 53, 54, 55, 58  
PRINT "A file problem has been encountered."  
CASE 57  
PRINT "An I/O fatal error has occurred."  
CASE 61  
PRINT "The disc receiving output is full."  
CASE 71, 72  
PRINT "There is no disc or it is flawed."  
CASE ELSE  
PRINT "An unidentified error has occurred."  
END SELECT  
LOCATE 20, 10  
PRINT "Any key to quit."  
a$ = readkey$  
COLOR 7, 0  
CLS  
END  

---SUBROUTINES---  
REM $STATIC  
SUB box (sd%, tr%, br%, lc%, rc%, btitle$, dr%)  
' Draws a single or doubled edged box in text mode.  
    IF (sd% = 1) THEN  
        RESTORE sline  
    ELSE  
        RESTORE dline  
    END IF  
    READ tl, tr, bl, br, hl, vl  
    wd% = rc% - lc% - 1  
    tl% = LEN(btitle$)  
    iok% = 1  
    IF (lc% < 1 OR rc% > 80) THEN iok% = 0  
    IF (tr% < 1 OR br% > 24) THEN iok% = 0  
    IF ((rc% - lc%) < 3) THEN iok% = 0  
    IF ((br% - tr%) < 2) THEN iok% = 0  
    IF (iok% = 0) THEN EXIT SUB  
    IF (dr% > 0) THEN  
        COLOR 4, 7  
        t$ = CHR$(t1) + STRING$(wd%, hl) + CHR$(tr)  
        m$ = CHR$(v1) + SPACES(wd%) + CHR$(v1)  
        bot$ = CHR$(b1) + STRING$(wd%, h1) + CHR$(br)  
    ELSE  
        COLOR 4, 11  
        wd% = wd% + 2  
        t$ = STRING$(wd%, 176)  
        m$ = t$: bot$ = t$  
        btitle$ = ""  
    END IF  
    r% = tr% - 1  
    ii% = br% - tr% + 1  
    FOR i% = 1 TO ii%  
        r% = r% + 1  
        LOCATE r%, lc%  
    END FOR  
    r% = tr%  
    ii% = br% - tr% + 1  
    FOR i% = 1 TO ii%  
        r% = r% + 1  
        LOCATE r%, lc% + 1  
    END FOR  
    r% = tr% - 1  
    ii% = br% - tr% + 1  
    FOR i% = 1 TO ii%  
        r% = r% + 1  
        LOCATE r%, lc% - 1  
    END FOR  
    r% = tr%  
    ii% = br% - tr% + 1  
    FOR i% = 1 TO ii%  
        r% = r% + 1  
        LOCATE r%, lc% + 1  
    END FOR  
    r% = tr% - 1  
    ii% = br% - tr% + 1  
    FOR i% = 1 TO ii%  
        r% = r% + 1  
        LOCATE r%, lc% - 1  
    END FOR  
    r% = tr%  
    ii% = br% - tr% + 1  
    FOR i% = 1 TO ii%  
        r% = r% + 1  
        LOCATE r%, lc% + 1  
    END FOR  
    r% = tr% - 1  
    ii% = br% - tr% + 1  
    FOR i% = 1 TO ii%  
        r% = r% + 1  
        LOCATE r%, lc% - 1  
    END FOR
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IF (i% = 1) THEN
  PRINT t$;
ELSEIF (i% = ii%) THEN
  PRINT bot$;
ELSE
  PRINT m$;
END IF
NEXT i%

IF (dr% > 0) THEN
  col% = lc% + CINT((wd% - tl%) / 2) + 1
  LOCATE tr%, col%
  PRINT btitle$;
END IF
END SUB

SUB calcs (ir%, e%, mtitle$)
  'Controls the calculation of inflated shapes.
  DIM lam(17), kurvc(3), xf(3) AS SINGLE
  delr = rad / nn
  IF (ir% = 1) THEN
    RESTORE lambda
    FOR i% = 1 TO 17
      READ lam(i%)
    NEXT i%
  END IF
  kp = 0 'zero profile count
  DO 'profile loop
    kp = kp + 1 'increment profile
    CALL box(1, 15, 10, 27, 54, ",", 1)
    COLOR 0
    LOCATE 16, 29
    PRINT "Calculations in Progress";
    LOCATE 17, 32
    PRINT USING "Profile number ##"; kp;
    IF (ir% = 1) THEN
      lamc = lam(kp) 'ext.ratio in range
    ELSEIF (ir% = 2) THEN
      lamc = maxext 'specified crown ext.ratio
    END IF
    laml = lamc: lam2 = lamc 'required ext.ratios
    CALL tension 'tension at crown
    tenc = ten2 'note tension at crown
    IF (kp = 1) THEN 'estimate for crown curv.
      kurvc(1) = 1 / rad 'first profile
    ELSE
      kurvc(1) = prevcurv(kp - 1) 'subsequent profiles
    END IF
    'Find upper and lower crown curvatures which straddle final X = rad.
    m% = 0 'zero iteration count
    cmodl = .8: cmod2 = 1.2 'set curvature modifiers
    DO
      m% = m% + 1 'iteration
      IF (m% > 10) THEN 'too many iterations
        profile = kp - 1
        e% = 1 'ignore current profile
      EXIT SUB
    END IF
    FOR j% = 1 TO 2
      kurv1 = kurvc(j%): kurv2 = kurvc(j%)
      pres = 2 * kurv1 * tenc 'internal pressure
      CALL prof(j%, lamc, tenc, xf(), e%)
      IF (e% = 3 AND e% = 4) THEN 'a calculation failure
        profile = kp - 1
        cmod1 = 1 - (1 - cmod1) / 2
        cmod2 = 1 + (cmod2 - 1) / 2
      END IF
    END FOR
END SUB
CALL diags(1, m%, j%, kurvc(), xf[])  'diagnostics file

' Modify pole curvature.
CALL modcurv(j%, kurvc(), xf[], cmod1, cmod2, flag%)
NEXT j%
LOOP UNTIL (flag% = 0)

' If necessary, swap limits to give increasing final radii.
IF (xf(1) > xf(2)) THEN
  sv = xf(1): xf(1) = xf(2): xf(2) = sv
  sv = kurvc(1): kurvc(1) = kurvc(2): kurvc(2) = sv
END IF

' Now interpolate to the required curvature.
  m% = 0
  DO
    m% = m% + 1
    IF (m% > 10) THEN
      e% = 2
      profile = kp - 1
      EXIT SUB
    END IF
    CALL interp(xf[], kurvc[])
    kurvl = kurvc(3): kurv2 = kurvc(3)
    pres = 2 * kurvl * tenc
    CALL prof(3, lamc, tenc, xf[], e%)
    IF (e% = 3 OR e% = 4) THEN
      profile = kp - 1
      EXIT SUB
    END IF
    CALL diags(2, m%, j%, kurvc[], xf[])
    CALL check(xfnd%)
    IF (xfnd% = 1) THEN
      prevcurv(kp) = kurvc(5)
      EXIT DO
    END IF
    CALL focus(xf[], kurvc[])
    CALL diags(3, m%, j%, kurvc[], xf[])
  LOOP

CALL stress(lamc, pstress)
CALL sed(lamc, energy)
CALL prnt(lamc, mtitle$, pstress, energy)
CALL test(ir%, finish%)
LOOP UNTIL (finish% = 1)
profile = kp
END SUB

SUB check (xfnd%)
' Checks the the final X coordinate for its closeness to rad.
  xfnd% = 0
  xr = rad
  IF (ABS((x - xr) / xr) < .001) THEN xfnd% = 1
END SUB

SUB clamp
' Draws the clamping ring (5 mm wide by 2 mm deep).
  xpl = rad: ypl = 0  'RHS
  xpr = rad + 5: ypr = -2
  LINE (xpl, ypl)-(xpr, ypr), 0, B
  xpr = -rad: ypr = 0  'LHS
  xpl = -rad - 5: ypl = -2
  LINE (xpl, ypl)-(xpr, ypr), 0, B
END SUB
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SUB diags (d%, m%, j%, kurvc(), xf())

' Writes to the diagnostics file at various stages of the calculations.
  fmt$ = "####.####"
  star$ = STRING$(37, "*")

' Initial curvature limits.
  IF (d% = 1) THEN
    IF (m% = 1 AND j% = 1) THEN
      CALL lines(3, 2)
      PRINT #3, TAB(10); "Profile number:"; kp; "
      PRINT #3, TAB(10); star$
    END IF
    IF (j% = 1) THEN
      CALL lines(3, 1)
      PRINT #3, TAB(10); "Initial limits:"
      PRINT #3, TAB(20); "Curvature: ";
      PRINT #3, USING fmt$; kurvc(j%)
      PRINT #3, TAB(45); "Final X coord: ";
      PRINT #3, USING fmt$; xf(j%)
    ELSE
      PRINT #3, TAB(29); ":
      PRINT #3, USING fmt$; kurvc(j%)
      PRINT #3, TAB(55); ":
      PRINT #3, USING fmt$; xf(j%)
    END IF
  END IF

' Results of interpolation.
  ELSEIF (d% = 2) THEN
    CALL lines(3, 1)
    PRINT #3, TAB(10); "Interpolated values:"
    PRINT #3, TAB(20); "Curvature: ";
    PRINT #3, USING fmt$; kurvc(3)
    PRINT #3, TAB(45); "Final X coord: ";
    PRINT #3, USING fmt$; xf(3)
  ELSEIF (d% = 3) THEN
    CALL lines(3, 1)
    PRINT #3, TAB(10); "Narrowed limits:"
    PRINT #3, TAB(20); "Curvature: ";
    PRINT #3, USING fmt$; kurvc(1)
    PRINT #3, TAB(45); "Final X coord: ";
    PRINT #3, USING fmt$; xf(1)
    PRINT #3, TAB(29); ":
    PRINT #3, USING fmt$; kurvc(2)
    PRINT #3, TAB(58); ":
    PRINT #3, USING fmt$; xf(2)
  END IF
END SUB

SUB dimens

' Enters the dimensions of the diaphragm and the number of annuli.
  LOCATE 9, 10
  PRINT "Diaphragm diameter (mm):"
  LOCATE 10, 10
  PRINT "Diaphragm thickness (mm):"
  LOCATE 11, 10
  PRINT "Number of annuli . . . 20, 30, 40 or 50:";

' Now enter values.
  DO
    FOR i% = 9 TO 10
      CLEAR OLD ENTRIES
    FOR i% = 9 TO 10
      PRINT SPACE$(5)
    NEXT i%
    PRINT SPACE$(5)
    NEXT i%
  END FOR
DO
  LOCATE i%, 36
  PRINT SPACE$(5); LOCATE i%, 36
  INPUT ",", cnst$
  cnst = datval!(cnst$, iok$)
LOOP UNTIL (iok$ = 1)
  IF (i% = 9) THEN
    rad = cnst / 2
  ELSE
    thk = cnst
  END IF
NEXT i%
DO
  LOCATE 12, 36
  PRINT SPACE$(5); LOCATE 12, 36
  INPUT ",", cnst$
  cnst = datval!(cnst$, iok$)
LOOP UNTIL (iok$ = 1)
  nn = CINT(cnst)
LOOP UNTIL ((nn \\ 10) * 10 = nn)
  ni = nn \\ 10
  a$ = affirm$(13, 10, "Correct")
LOOP UNTIL (a$ = "y")
  CALL box(1, 5, 14, 8, 42, ",", 0)
END SUB
SUB display (ch%, mtitle$)

  'controls the display of profiles.
  SCREEN 12
  COLOR 14
  CLS
  PAINT (100, 100), 1
  LOCATE 3, 8
  PRINT "RuPEC \t INFLATED DIAPHRAGM \t RuPEC"
  LOCATE 5, 8
  PRINT mtitle$;
  IF (ch% = 1) THEN
    LOCATE 5, 46
    PRINT "Stepped Display"
  ELSE
    LOCATE 5, 45
    PRINT "Kinematic Display"
  END IF

  'create a view port and define physical co-ordinates (mm).
  dxmax = 2 * xmax
  IF (dxmax > ymax) THEN
    rng = dxmax
  ELSE
    rng = ymax
  END IF
  apres(0) = 0
  VIEW (240, 100)-(600, 460), 11, 14
  tlx = -1.1 * rng / 2; tly = 1.1 * rng
  brx = 1.1 * rng / 2; bry = -.1 * rng
  WINDOW (tlx, tly)-(brx, bry)
  IF (ch% = 1) THEN
    DO
      LOCATE 28, 4
      PRINT "Any key for next."
      FOR kk% = I TO 2
        IF (kk% = 1) THEN
          prs% = 0; prf% = profile; prb% = 1
        ELSE
          prs% = profile; prf% = 0; prb% = -1
        END IF
        FOR k% = prs% TO prf% STEP prb%
          LOCATE 10, 4
          PRINT ";"
        END FOR
      END FOR
    END DO
  END IF

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PRINT USING "Profile Number #"; k%
LOCATE 12, 4
PRINT USING "Pressure = ####.#### kPa"; apres(k%)
call show(k%, 1) 'draw
call clamp 'draw clamp ring
a$ = readkeys$
call show(k%, 2) 'erase
call clamp 'draw clamp ring
next k%
next k%
a$ = affirm$(28, 4, "Another cycle")
locate 28, 4
print spaces$(25);
loop until (a$ = "n") 'kinematic display
dt = 1 / profile 'delay time
locate 10, 4
print using "Profiles 0 to #"; profile
locate 12, 4
print using "Pressures 0 to ####.#### kPa"; apres(profile)
do
for k% = 0 to profile 'to maximum height
call show(k%, 1) 'draw
call clamp 'draw clamp ring
call delay(dt) 'time delay
call show(k%, 2) 'erase
call clamp 'draw clamp ring
next k%
for k% = profile to 0 step -1 'to rebound
call show(k%, 1) 'draw
call clamp 'draw clamp ring
call delay(dt) 'time delay
call show(k%, 2) 'erase
call clamp 'draw clamp ring
next k%
loop until (inkeys$ = "x" or inkeys$ = "X") 'to stop
end if
screen 0
call surround(1)
end sub

' Terminates the program.
call surround(2)
color 0
if (done(7) = 0 or e% = -1) then 'calculations not made
locate 12, 30
print "RUN ABORTED."
locate 16, 10
else 'calculations made
if (e% = 0) then 'all calculations OK
locate 12, 27
print "RUN SUCCESSFUL."
else if (profile = 0) then 'complete failure
locate 12, 25
print "RUN UNSUCCESSFUL."
else 'partial failure
locate 12, 20
print "RUN PARTLY SUCCESSFUL."
end if
end if
p% = profile + 1 'reason for failure
locate 14, 12
if (e% = 1) then 'Profile ## - curvature limits not found.
p% = profile + 1
elseif (e% = 2) then 'Profile ## - curvature interpolation failed.
p% = profile + 1
elseif (e% = 3) then 'Profile ## - Turner lambda iteration failed.
p% = profile + 1
elseif (e% = 4) then 'Profile ## - Pressure iteration failed.
p% = profile + 1
end if
end if
locate 16, 12
print "The output files are in directory: "; working; ";.";
LOCATE 17, 17
PRINT "Any printed results are in the file "; basename; ".prt";
LOCATE 18, 17
PRINT "Summary results are in file " ; basename; ".sum";
LOCATE 19, 17
PRINT "The Excel file is "; basename; ".csvi";
END IF
LOCATE 22, 12
PRINT "Any key to continue."
a$ = readkey$
CALL surround(1)
END SUB

SUB fillarray (i%)
' Completes the master arrays as the calculations proceed.

xc(i%) = x; yc(i%) = y
ang(i%) = a
curv(1, i%) = kurv1; curv(2, i%) = kurv2
ten(1, i%) = ten1; ten(2, i%) = ten2
ext(1, i%) = lam1
ext(2, i%) = lam2
ext(3, i%) = lam3
END SUB

SUB focus (xf(), kurvc())
' Narrows the limits when interpolating the crown curvature.

xr = rad
IF (xf(3) < xr) THEN 'final X coord. too small
kurvc(1) = kurvc(3)
xf(1) = xf(3)
ELSE 'final X coord. too large
kurvc(2) = kurvc(3)
xf(2) = xf(3)
END IF
END SUB

SUB frame
' Creates a running frame around the opening screen.

SCREEN 7
COLOR 14, 1
line$ = CHR$(201) + STRING$(35, 205) + CHR$(187)
LOCATE 2, 2
PRINT line$: 'top
FOR i% = 3 TO 23
line$ = CHR$(186)
LOCATE i%, 2
PRINT line$;
NEXT i%
line$ = CHR$(200) + STRING$(36, 205) + CHR$(188)
LOCATE 24, 2
PRINT line$;
LOCATE 22, 4
PRINT "JFH/PSO";
END SUB

SUB interp (xf(), kurvc())
' Interpolates a value for the crown curvature between limits.

xr = rad
dkurv = kurvc(2) - kurvc(1)
exf = xf(2) - xf(1)
kurvc(3) = kurvc(1) + (xr - xf(1)) * dkurv / dxf
END SUB
SUB menu (men%, sd%, bttitle$, tr%, lc%, sel%)
'Reads required menu list, displays the menu and allows selection.
'See RUPEC.BAS

END SUB

SUB modconsts (name$)
'Enters the elastic constants of the modified Turner function.

DIM prompt$(7)
name$ = "-" 'default compound ref.
prompt$(1) = " Initial tension:"
prompt$(2) = " Linear coefficient:"
prompt$(3) = " Quadratic coefficient:"
prompt$(4) = " Cubic coefficient:"
prompt$(5) = " Quartic coefficient:"
prompt$(6) = " Quintic coefficient:"
prompt$(7) = " Equibiaxial factor:"
nt% = 7
br% = 7 + nt%
CALL box(1, 5, br%, 8, 42, " MODIFIED TURNER CONSTANTS ", 1)
COLOR 0

col% = 33
FOR i% = 1 TO nt%
LOCATE 5 + i%, col%
PRINT prompt$(i%)
NEXT i%

'Display constant prompts
FOR i% = 1 TO nt%
DO
LOCATE 5 + i%, col%
INPUT ", cnst$
ctnst(i%) = datval!(cnst$, iok%)
LOOP UNTIL (iok% = 1)
NEXT i%

a$ = affirm$(br% - 1, 10, "Correct")
LOOP UNTIL (a$ = "y")
CALL box(1, 5, br%, 8, 42, "", 0)
END SUB

SUB modcurv (j%, kcurv(), xf(), cmodl, cmod2, flag%)
'Modifies the crown curvature according to the previous final X coordinate.

xr = rad

IF (j% = 1) THEN
IF (xf(1) < xr) THEN 'decrease curvature
kcurv(2) = cmodl * kcurv(1)
flag% = -1
ELSEIF (xf(1) > xr) THEN 'increase curvature
kcurv(2) = cmod2 * kcurv(1)
flag% = 1
END IF
END IF

IF (j% = 2) THEN
IF (flag% = -1) THEN
IF (xf(2) > xr) THEN 'straddle found
flag% = 0
END IF
END IF
ELSE
    kurvc(1) = kurvc(2)
    try again
END IF

ELSEIF (flag$ = 1) THEN
    IF (xf(2) < xr) THEN
    straddle found
    flag$ = 0
    ELSE
    kurvc(1) = kurvc(2)
    try again
    END IF
    END IF
END IF
END IF
END SUB

SUB modF (name$)
    'Enters just the Equibiaxial Factor
    DIM prompt$(5)
    name$ = "-"
    prompt$(1) = "Equibiaxial factor:"
    nt% = 1
    br% = 7 + nt%
    CALL box(1, 5, br%, 8, 39, " ORIGINAL TURNER CONSTANTS ", 1)
    COLOR 0
    col% = 30          'column for data
    FOR i% = 1 TO nt%
        LOCATE 5 + i%, 10
        PRINT prompt$(i%)
    NEXT i%

    'Now enter or display values for the required constants.
    DO
        FOR i% = 1 TO nt%
            LOCATE 5 + i%, col%
            PRINT SPACE$(6);          'display constant prompts
        NEXT i%
        LOCATE br% - 1, 10
        PRINT SPACE$(20)
        FOR i% = 1 TO nt%
            DO
                LOCATE 5 + i%, col%
                PRINT SPACE$(6);          'original Turner parameters
                PRINT SPACE$(6);          'and equibiaxial factor
                LOCATE 5 + i%, col%
                INPUT ", cnst$
                cnst = datval!(cnst$, iok%)
                LOOP UNTIL (iok% = 1)
                tconst(4 + i%) = cnst
            NEXT i%
    a$ = affirm$(br% - 1, 10, "Correct")
    LOOP UNTIL (a$ = "y")
    CALL box(1, 5, br%, 8, 39, ",", 0)
END SUB

SUB origconsts (name$)
    'Enters the elastic constants of the original Turner function.
    DIM prompt$(5)
    name$ = "-"
    prompt$(1) = "Initial tension:"
    prompt$(2) = "Linear coefficient:"
    prompt$(3) = "Power coefficient:"
    prompt$(4) = "Power index:"
    prompt$(5) = "Equibiaxial factor:"
    nt% = 5
    br% = 7 + nt%
    CALL box(1, 5, br%, 8, 39, " ORIGINAL TURNER CONSTANTS ", 1)
    COLOR 0
col% = 30
FOR i% = 1 TO nt%
  LOCATE 5 + i%, 10
  PRINT promptS(i%)
NEXT i%

'Now enter or display values for the required constants.
DO
  FOR i% = 1 TO nt%
    LOCATE 5 + i%, col%
    PRINT SPACES(6);  
    NEXT i%
  FOR i% = 1 TO nt%
    DO
      LOCATE 5 + i%, col%
      PRINT SPACES(6);  
      LOCATE 5 + i%, col%
      INPUT "", cnst$
      cnst = datval!(cnst$, iok%)
      LOOP UNTIL (iok% = 1)
      tcnst(i%) = cnst
    NEXT i%
  a$ = affirm$(br% - 1, 10, "Correct")
  LOOP UNTIL (a$ = "y")
END SUB

SUB plot (ir%, mtitle$)
'Plots the profiles on the screen.
  DIM sxx(51), syy(51)
  xmax = 0: ymax = 0
  ii% = nn + 1
  FOR k% = 1 TO profile  
    'save in temporary arrays
    FOR i% = 1 TO ii%
      sxx(i%) = xx(k%, i%)
      syy(i%) = yy(k%, i%)
      IF (sxx(i%) > xmax) THEN xmax = sxx(i%)  
      IF (syy(i%) > ymax) THEN ymax = syy(i%)  
      NEXT i%
    FOR i% = 1 TO ii%
      'rewrite in reverse order
      j% = ii% - i% + 1
      xx(k%, j%) = sxx(i%)
      yy(k%, j%) = syy(i%)
      j% = ii% + i%
      xx(k%, j%) = -sxx(i%)
      yy(k%, j%) = syy(i%)
      NEXT i%
  NEXT k%
  'Now choose type of display.
  br% = 12 - ir%
  DO
    CALL box(1, 5, br%, 8, 39, "DISPLAY ", 1)
    COLOR 0
    LOCATE 7, 12
    PRINT "Choose from:"
    IF (ir% = 1) THEN
      LOCATE 8, 12
      PRINT "2. Kinematic display;"
    ELSE IF (ir% = 0) THEN
      LOCATE 8, 12
      PRINT "0. For summary."
    ELSE
      LOCATE 11 - ir%, 12
      PRINT "DISPLAY ", 1)
    END IF
  DO
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IF (ir% = 1) THEN
    INPUT "Enter 0 to 2: ", ch$
ELSE
    INPUT "Enter 1 or 0: ", ch$
END IF

IF (ch$ <> "") THEN ch% = ASC(ch$) - 48
LOOP UNTIL (ch% >= 0 OR ch% < 3)

IF (ch% = 0) THEN
    EXIT SUB
ELSE
    CALL display(ch%, mtitle$)
END IF
LOOP
END SUB

SUB prntout (ir%)
    'Allows suppression of displayed tables and results file.
    DO
        disp = 0: pfile = 0
        CALL box(1, 5, br%, 8, 38, ", 0)
        COLOR 0
        IF (ir% = 1) THEN
            a$ = affirm$(6, 10, "Display tabulated results")
            LOCATE 6, 10
            PRINT "Display tabulated results? - y/n: ">
            IF (a$ = "y") THEN disp = 1
            ELSE
disp = 1
        END IF
        a$ = affirm$(7, 10, " File tabulated results")
        IF (a$ = "y") THEN pfile = 1
        a$ = affirm$(8, 10, "Correct")
        LOOP UNTIL (a$ = "y")
        CALL box(1, 5, 9, 8, 47, ", 0)
    END SUB

SUB prof (j%, lamc, tenc, xf(), e%)
    'Generates a profile using incremental arcs and based upon the Turner function.
    e% = 0
    lam1 = lamc: lam2 = lamc: lam3 = 1 / (lamc * lamc)
    ten1 = tenc: ten2 = tenc

    'Initialise coordinates and meridional angle at the pole,
    'together with the sine and cosine of the angle.
    x = 0: y = 0: a = 0
    sins = 0: cosa = 1
    i% = 1

    DO
        CALL fillarray(i%)
        i% = i% + 1
        IF (lam2 <= 1 OR i% = nn + 2) THEN EXIT DO
        da = lam1 * delr * kurvl
        a = a + da
        dx = (SIN(a) - sina) / kurvl
        x = x + dx
        dy = (COS(a) - cosa) / kurvl
        y = y + dy
        lam2 = x / ((i% - 1) * delr)
    LOOP

    'Iterate to determine the new lambda 1, based on matching the pressure.
    m% = 0
    DO
        m% = m% + 1
        IF (m% > 100) THEN
            e% = 3
            EXIT SUB
    LOOP
'lambdas ...

END IF
CALL tension
npres = 2 * tenl * SIN(a) / x'principal tensions
IF (pres < 0 OR npres < 0) THEN
   e% = 4'pressures match?
EXIT SUB
END IF
lam1 = lam1 * (pres / npres) ^ .3
LOOP UNTIL ((ABS(pres / npres) - 1) < .001)'adjust lambda 1
kurv2 = pres / (2 * tenl) 'new circum.curvature
kurv1 = kurv2 * (2 - ten2 / tenl) 'new radial curvature
sina = SIN(a) 'for next increment
cosa = COS(a)
LOOP
xf(j%) = x 'for interpolation etc.
END SUB

SUB range (ir%)
 'Enter a value for the range of maximum extension ratios or
'a specific maximum extension ratio at the crown.

   IF (ir% = 1) THEN
      CALL box(1, 5, 9, 8, 37, " MAXIMUM HEIGHT ", 1)
   ELSE
      CALL box(1, 5, 9, 8, 37, " EXT.RATIO (Max.of 10) ", 1)
   END IF
COLOR 0
   DO
      LOCATE 6, c%
      PRINT "Enter the required":
      LOCATE 7, 10
      PRINT "maximum height (mm)":
      c% = 31
      ELSE
         LOCATE 6, 10
         PRINT "Enter the specific":
         LOCATE 7, 10
         PRINT "extension ratio":
         c% = 27
   END IF
   DO
      LOCATE 7, c%
      PRINT SPACE$(6)
      LOCATE 7, c%
      INPUT ", max$
      IF (ir% = 1) THEN
         maxht = datval!(max$, iok%)
      ELSE
         maxext = datval!(max$, iok%)
         IF (maxext > 10) THEN iok% = 0
      END IF
      LOOP UNTIL (iok% = 1)
      a$ = affirm$(8, 10, "Correct")
      a$ = affirm$(8, 10, "Correct")
   END DO
   CALL box(1, 5, 9, 8, 37, ", 0)
   END SUB

SUB sed (lamc, energy)
 'Calculates the strain energy densities at the poles from
'the integral "f.de" using the data from the original or
'modified Turner function.

   lam1 = lamc: lam2 = lam1 'pole lambdas
   CALL tension
   engl = tenl / (lam1 * lam3 * thk) 'pole tensions
   stnl = lam1 - 1 'engineering stress 1
   IF (kp = 1) THEN
     energy = engl * stnl / 2 'strain energy
   ELSE
     eng2 = pstr(kp - 1) / lamax(kp - 1) 'previous eng.stress
   END IF

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\[
\text{stn2} = \text{lamax}(kp - 1) - 1 \\
\text{se} = (\text{engl} + \text{eng2}) \times (\text{stn1} - \text{stn2}) / 2 \\
\text{energy} = \text{psed}(kp - 1) / 2 + \text{se} \\
\end{array}
\]

END IF

energy = energy * 2

END SUB

SUB show (k%, dd%)

'Displays a complete diaphragm profile.

IF (dd% = 1) THEN
  c% = 0
ELSE
  c% = 11
END IF

IF (k% = 0) THEN
  xp = rad: yp = 0
  PSET (xp, yp), c%
  xp = -rad
  LINE -(xp, yp), c%
ELSE
  FOR i% = 1 TO 2 * nn + 2
    xp = xx(k%, i%)
    yp = yy(k%, i%)
    IF (i% = 1) THEN
      PSET (xp, yp), c%
    ELSE
      LINE -(xp, yp), c%
    END IF
  NEXT i%
END IF

ty = 1.05 * mg: by = -.05 * mg
LINE (0, by)-(0, ty), 0" sHFFCC

END SUB

SUB specify (mtitle$)

'Enters the descriptive title for the simulation (maximum of 40 ch.s).

DO
  CALL box(1, 5, 14, 8, 42, " DIAPHRAGM SPECIFICATION ", 1)
  COLOR 0
  LOCATE 6, 10
  PRINT "Title (max.of 20 ch.):";
  mtitle$ = atitle$(7, 15, 20)
  a$ = affirm$(8, 10, "Correct")
  LOOP UNTIL (a$ = "y")

'Write to print file.

PRINT #3, TAB(10); "RuPEC" \
PRINT #3, TAB(10); "Date: "; newdate \
CALL lines(3, 2) \
PRINT #3, TAB(19); "DIAGNOSTICS - INFLATED CIRCULAR DIAPHRAGM." \
PRINT #3, TAB(19); "-----------------------------" \
CALL lines(3, 2) \
PRINT #3, TAB(10); "Title: "; mtitle$ 

IF count > 1 THEN
  a$ = affirm$(9, 10, "New Dimensions")
  '2nd run
  IF a$ = "y" THEN
    LOCATE 9, 10: PRINT SPACE$(20)
    'clear questions
    CALL dimens
  ELSE
    CALL box(1, 5, 14, 8, 42, "", 0)
  END IF
  ELSE
    CALL dimens
  END IF

END SUB

SUB stress (lamc, pstress)

'Calculates the true principal stress at the pole.

lam1 = lamc: lam2 = lamc: lam3 = 1 / (lamc * lamc)
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length = SQR(laml * laml + lam2 * lam2 + lam3 * lam3) 'diagonal length
extn = (length - lengo) / lengo 'diagonal strain
alf1 = ATN(SQR(lam2 * lam2 + lam3 * lam3) / laml) 'angles
alf2 = ATN(SQR(lam3 * lam3 + laml * laml) / lam2)
alf3 = ATN(SQR(laml * laml + lam2 * lam2) / lam3)

tens = turner!(extn) 'tension
forcl = tens * COS(alf1) 'forces
forc2 = tens * COS(alf2) 'or
forc3 = tens * COS(alf3) 'eng.stresses

strs1 = forcl * laml 'true stresses
strs2 = forc2 * lam2
strs3 = forc3 * lam3

'Correct for hydrostatic pressure.
pstress = strs1 - strs3 'actual true stress at crown

END SUB

SUB surround (back%)
'Draws a surrounding frame on the screen.

RESTORE dline
READ ti, tr, bl, br, hl, vi 'surround components
IF (back% = 1) THEN
p$ = CHR$(176) 'background pattern
ELSE
p$ = CHR$(32) 'plain
END IF
txt$ = “RuPEC”
COLOR 4, 11
CLS
m$ = p$ + CHR$(v1) + STRING$(76, p$) + CHR$(v1) + p$ 'sides
b$ = PS + CHR$(b1) + STRING$(76, hl) + CHR$(br) + p$ 'bottom

t$ = p$ + CHR$(t1) + STRING$(76, hl) + CHR$(tr) + IDS 'top of box
m$ = p$ + CHR$(vl) + STRING$(76, p$) + CHR$(vl) + p$ 'sides
b$ = p$ + CHR$(bl) + STRING$(76, hl) + CHR$(br) + p$ 'bottom

LOCATE 1, 1 PRINT t$;
LOCATE 1, 30 PRINT "INFLATED DIAPHRAGM"
FOR i% = 2 TO 24
LOCATE i%, 1 PRINT m$;
NEXT i%
LOCATE 25, 1 PRINT b$;

LOCATE 1, 6: PRINT txt$;
LOCATE 1, 71: PRINT txt$;
LOCATE 25, 6: PRINT txt$;
LOCATE 25, 71: PRINT txt$:
END SUB

SUB tension
'Calculates both principal tensions at a point.

lam3 = 1 / (laml * lam2) 'lambda 3

length = SQR(laml * laml + lam2 * lam2 + lam3 * lam3) 'diagonal length
extn = (length - lengo) / lengo 'diagonal strain
alf1 = ATN(SQR(lam2 * lam2 + lam3 * lam3) / laml) 'angles
alf2 = ATN(SQR(lam3 * lam3 + laml * laml) / lam2)
alf3 = ATN(SQR(laml * laml + lam2 * lam2) / lam3)

tens = turner!(extn) 'tension
forcl = tens * COS(alf1) 'forces
forc2 = tens * COS(alf2) 'or
forc3 = tens * COS(alf3) 'eng.stresses

strs1 = forcl * laml 'true stresses
strs2 = forc2 * lam2
strs3 = forc3 * lam3

'Correct for hydrostatic pressure and convert to tensions per unit width.
stress1 = strs1 - strs3  
stress2 = strs2 - strs3  
ten1 = stress1 * lam3 * thk  
ten2 = stress2 * lam3 * thk

SUB test (ir%, finish%)  
'Tests whether the required height has been exceeded.  
finish% = 0  
IF (ir% = 1) THEN  
   IF (ht(kp) > maxht) THEN  
      finish% = 1  
   ELSE  
      IF (kp = 17) THEN finish% = 1  
   END IF  
ELSE  
   finish% = 1  
END IF

FUNCTION turner! (extn)  
'Calculates the tension in the elastic member,  
'allowing for biaxial adjustment.  
IF (mt = 1) THEN  
   tens = tcnst(1) + tcnst(2) * extn + tcnst(3) * extn * tcnst(4)  
ELSEIF (mt = 2) THEN  
   tens = tcnst(1)  
   FOR i% = 2 TO 6  
      tens = tens + tcnst(i%) * (extn * (i% - 1))  
   NEXT i%  
END IF

dlam = ABS(laml - lam2)  
IF (laml > lam2) THEN  
   damax = laml - 1 / SQR(laml)  
ELSE  
   damax = lam2 - 1 / SQR(lam2)  
END IF

IF (mt = 1) THEN  
   factor = tcnst(5) + dlam * (1 - tcnst(5)) / damax  
ELSEIF (mt = 2) THEN  
   factor = tcnst(7) + dlam * (1 - tcnst(7)) / damax  
END IF

turner! = factor * tens  
END FUNCTION

SUB volume  
'Calculates the contained volume.  
vol(kp) = 0  
FOR i% = 1 TO nn  
   avx = (xc(i%) + xc(i% + 1)) / 2  
   dy = yc(i%) - yc(i% + 1)  
   vol(kp) = vol(kp) + pi * avx * avx * dy  
NEXT i%

END SUB

SUB work  
'Calculates the total stored energies by the integration "p.dv".  
FOR k% = 1 TO profile  
   IF (k% = 1) THEN  
      twork(k%) = vol(k%) * apres(k%) / 2  
   ELSE  
      de = (apres(k%) + apres(k% - 1)) * (vol(k%) - vol(k% - 1)) / 2  
      twork(k%) = twork(k% - 1) + de  
   END IF  
NEXT k%

FOR k% = 1 TO profile  
   twork(k%) = twork(k%) / 1000000  
END SUB
FUNCTION affirm$ (ln%, col%, text$)

'Accepts a question and returns a "y/n" answer.
'See RUPEC.BAS

END FUNCTION

FUNCTION atitle$ (ln%, col%, tl%)

'Enters a title with a maximum number of characters, with:
'  ln% = line;  col% = column of first character entered;
'  tl% = maximum length of the title.
'Note: > and < are displayed at either end of the input zone.
'See RUPEC.BAS

END FUNCTION

SUB choose (men%, items%, keys, sel%)

'Allows a menu item to be selected.
'See RUPEC.BAS

END SUB

SUB datemod

'Changes the internal date format (mm-dd-yyyy) to (dd mon yyyy).
'See RUPEC.BAS

END SUB

FUNCTION datval! (vnum$, iok%)

'Accepts a string representing a number, checks its validity
'and returns the number, with:
'  vnum$ = the string;  vnum = the required number;
'  iok% = 0 if the string is invalid, otherwise iok% = 1.
'See RUPEC.BAS

END FUNCTION

SUB delay (dt)

'Provides a delay of dt seconds.
'See RUPEC.BAS

END SUB

FUNCTION drive$ (ln%, col%)

'Returns a disc drive letter, A,B,C or D.
'See RUPEC.BAS

END FUNCTION

SUB excel

'Sends the bulk of the summary results to the Excel file 'diaph.csv'.

PRINT #4, "Diameter,Thickness,Increments"
PRINT #4, USING "###.##", 2 * rad;
PRINT #4, USING "##.###,"; thk;
PRINT #4, USING "###"; nn
PRINT #4, "Lambda,Pressure,Height,Width,At height,Curvature,";
PRINT #4, "Stress,S.E.D.,Major axis,Minor axis,Volume,Energy"
FOR k% = 1 TO profile
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SUB fileconsts (mtitle$, name$)
'Outputs the elastic constants to the full print file.
fmt$ = "##.###^^^^"
PRINT #1, CHR$(12) 'new page
PRINT #1, TAB(5); "RuPEC"; TAB(60); "Date: "; newdate
CALL lines(1, 2)
PRINT #1, TAB(28); "INFLATED CIRCULAR DIAPHRAGM."
PRINT #1, TAB(28); "---------------------------"
CALL lines(1, 2) 'titles
PRINT #1, TAB(10); "Title: "; mtitle$
PRINT #1, TAB(10); "Data file (.tnr) reference: "; name$
CALL lines(1, 1)
PRINT #1, TAB(10); "Elastic Constants (MPa):"
CALL lines(2, 1)
'Modified Turner parameters.
CALL lines(2, 1)
PRINT #1, TAB(15); "Initial tension: ";
PRINT #1, USING fmt$; tcnst(1);
PRINT #1, TAB(49); "Linear coeff.: ";
PRINT #1, USING fmt$; tcnst(2)
PRINT #1, TAB(15); "Quadratic coeff.: ";
PRINT #1, USING fmt$; tcnst(3)
PRINT #1, TAB(15); "Cubic coeff.: ";
PRINT #1, USING fmt$; tcnst(4)
PRINT #1, TAB(15); "Quartic coeff.: ";
PRINT #1, USING fmt$; tcnst(5)
PRINT #1, TAB(15); "Quintic coeff.:
PRINT #1, USING fmt$; tcnst(6)
PRINT #1, TAB(15); "Equibiax.factor: ";
PRINT #1, USING "##.###"; tcnst(7)
'(Diaphragm dimensions.
CALL lines(1, 2)
PRINT #1, TAB(10);
PRINT #1, USING "Diaphragm diameter = ##.## mm"; 2 * rad
PRINT #1, TAB(10);
PRINT #1, USING "thickness = ##.## mm"; thk
PRINT #1, TAB(10);
PRINT #1, USING "No. of increments = "; nn
CALL lines(1, 2)
star$ = STRING$(38, "'
PRINT #1, TAB(23); star$
END SUB

SUB filedat (p%, lamc, mtitle$, pstress, energy)
'Files the complete results for subsequent printing.
ii% = nn + 1
star$ = STRING$(39, "*"
IF (p% = 1) THEN 'part one
PRINT #1, CHR$(12)
PRINT #1, TAB(5); "RuPEC"; TAB(60); "Date: "; newdate
CALL lines(1, 2)
PRINT #1, TAB(28); "INFLATED CIRCULAR DIAPHRAGM."
PRINT #1, TAB(28); "---------------------------"
CALL lines(1, 2)
PRINT #1, TAB(10); "Title: "; mtitle$
PRINT #1, TAB(10); "Profile details.
kp$ = LTRIM$(STR$(kp))
PRINT #1, TAB(10); "Profile Number: "; kp$
PRINT #1, TAB(10); "Extension Ratio at the Pole: ";
PRINT #1, USING "##.##"; lamc
PRINT #1, TAB(10); "Internal Pressure: ";
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PRINT #1, USING "###.#### kPa"; apres(kp)
CALL lines(1, 2)

'Coordinates of the profile.
PRINT #1, TAB(10); " X  Y  Angle  "
PRINT #1, "Radial  Circum"
PRINT #1, TAB(10); " Coord  Coord  (deg)  "
PRINT #1, "Radius  Radius"
PRINT #1, TAB(10); " (mm)  (mm)  "
PRINT #1, "(mm)  (mm)"
CALL lines(1, 1)

fmt1$ = "####.####  ####.####  ####.####  ####.####"
fmt2$ = "####.####  ####.####  ####.####  ####.####"
FOR i% = 1 TO ii% STEP ni
PRINT #1, USING fmt1$; xc(i%); yc(i%); ang(i%);
PRINT #1, USING fmt2$; curv(1, i%); curv(2, i%)
NEXT i%
CALL lines(1, 1)
PRINT #1, TAB(20); star$

'Tensions and extension ratios around the profile.
ELSE 'part two
CALL lines(1, 3)
PRINT #1, TAB(10); " Radial  Circum  Thick  "
PRINT #1, "Radial  Circum"
PRINT #1, TAB(10); " Lambda  Lambda  Lambda  "
PRINT #1, "Tension  Tension"
PRINT #1, TAB(48); "(kN/m)  (kN/m)"
CALL lines(1, 1)

fmt1$ = "##.####  #.####
fmt2$ = "##.####  #.####
FOR i% = 1 TO ii% STEP ni
PRINT #1, USING fmt1$; ext(1, i%); ext(2, i%); ext(3, i%);
PRINT #1, USING fmt2$; ten(1, i%); ten(2, i%)
NEXT i%

'Conditions at the pole.
CALL lines(1, 1)
PRINT #1, TAB(10);
PRINT #1, USING "True stress at the pole = ###.#### MPa"; pstress
PRINT #1, USING "S.E.D at the Pole = ###.#### MJ/m^3"; energy
CALL lines(1, 1)
PRINT #1, TAB(20); star$
CALL lines(1, 3)
END IF
END SUB

SUB fopen (f%)
'Opens a file after requesting the drive and directory/sub-directory.
'See RUPEC.BAS
END SUB

SUB headframe (tr%, lc%, wd%, dp%)
'Draws a simple box around headings.
'See RUPEC.BAS
END SUB

SUB lines (f%, nl%)
'Outputs blank lines to the print file.
'See RUPEC.BAS
END SUB
SUB modconfile (name$)
' Reads the elastic constants of the modified Turner function from a file.
  CALL fopen(3)
  INPUT #5, name$
  INPUT #5, dum$
  FOR i% = 1 TO 6
    INPUT #5, tcnst(i%)
  NEXT i%
  CLOSE #5
END SUB

FUNCTION ok$
' Returns a 'y' or 'n' answer. Other, invalid answers must be trapped 'in the calling subroutine, OR is assumed yes.
' See RUPEC.BAS
END FUNCTION

SUB origconfile (name$)
' Reads the elastic constants of the original Turner function from a file.
  CALL fopen(2)
  INPUT #5, name$
  INPUT #5, dum$
  FOR i% = 1 TO 4
    INPUT #5, tcnst(i%)
  NEXT i%
  CLOSE #5
END SUB

SUB prnt (lamc, mtitle$, pstress, energy)
' Displays the coordinates of a profile.
  deg = 180 / (4 * ATN(1))   'angle conversion
  ii% = nn + 1               'number of points
  apres(kp) = 1000 * pres    'actual pressure (kPa)
  lamax(kp) = lamc           'for summary screen
  pstr(kp) = pstress         'for summary screen
  psed(kp) = energy          'for summary screen
  FOR p% = 1 TO 2
    IF (disp = 1) THEN
      CALL surround(2)
      COLOR 0
      LOCATE 4, 11
      PRINT USING "Profile Ili"; kp
      LOCATE 4, 26
      PRINT USING "Pole Lambda = 0.####"; lamc
      LOCATE 4, 51
      PRINT USING "Pressure = 0###.# kPa"; apres(kp)
    END IF
  END IF
  'Modify dimensions and seek maximum width.
  IF (p% = 1) THEN
    maxwd = 0
    FOR i% = 1 TO ii%
      IF (xc(i%) > maxwd) THEN
        maxwd = xc(i%)
        kw% = i%
      END IF
    NEXT i%
    yc(i%) = yc(i%) - yc(ii%)
    ang(i%) = ang(i%) * deg
    FOR j% = 1 TO 2
      curv(j%, i%) = 1 / curv(j%, i%)
    NEXT j%
 IF (disp = 1) THEN
    LOCATE 6, 11
    PRINT " X Y Angle Radial Circum"
    LOCATE 7, 11
    PRINT " Coord Coord (deg) Radius Radius"
    LOCATE 8, 11
    PRINT " (mm) (mm) (mm) (mm)"
    fmt$ = " ####.# ####.# ####.# ####.#"
    fmt2$ = "##.## ##.## ##.## ##.##"
    j% = 9
    FOR i% = 1 TO ii% STEP ni
        j% = j% + 1
        LOCATE j%, 11
        PRINT USING fmt$; xc(i%); yc(i%); ang(i%); curv(1, i%); curv(2, i%)
    NEXT i%
    END IF
    IF (pfile = 1) THEN CALL filedat(p%, lamc, mtitle$, pstress, energy)
    END IF
END IF
ELSE
    IF (disp = 1) THEN
        LOCATE 6, 11
        PRINT " Radial Circum Thick Radial Circum"
        LOCATE 7, 11
        PRINT " Lambda Lambda Lambda Tension Tension"
        LOCATE 8, 11
        PRINT " (kN/m) (kN/m)"
        fmt1$ = "###.### ###.### ###.### ###.###"
        fmt2$ = "##.## ##.## ##.## ##.##"
        j% = 9
        FOR i% = 1 TO ii% STEP ni
            j% = j% + 1
            LOCATE j%, 11
            PRINT USING fmt1$; ext(1, i%); ext(2, i%); ext(3, i%);
            PRINT USING fmt2$; ten(1, i%); ten(2, i%)
        NEXT i%
        LOCATE 22, 35
        PRINT USING "True stress at pole = #.####——MPa"; pstress
        LOCATE 23, 35
        PRINT USING "SED at pole = #.####——MJ/m^3"; energy
        END IF
    IF (pfile = 1) THEN CALL filedat(p%, lamc, mtitle$, pstress, energy)
    END IF
    IF (disp = 1) THEN
        LOCATE 23, 10
        PRINT "Any key to continue."
        a$ = readkey$
    END IF
NEXT p%
END SUB
SUB prntsum (mtitle$, name$)
    'Sends the summary data to the print file 'ndiaph.sum'.
    IF (count > 1) THEN PRINT #2, CHR$(12)
    p$ = LTRIM$(STR$(profile))
    PRINT #2, TAB(30); "SUMMARY OF "; p$; " PROFILES."
    CALL sumconsts(mtitle$, name$) 	 'file constants
    CALL lines(2, 1)
    PRINT #2, MAB(10); "Diaphragm diameter = ##.## mm";
    PRINT #2, TAB(10); "True stress at pole = #.####——MPa"; pstress
    PRINT #2, TAB(10); "SED at pole = #.####——MJ/m^3"; energy
    END IF
    IF (pfile = 1) THEN CALL filedat(p%, lamc, mtitle$, pstress, energy)
    END IF
    IF (disp = 1) THEN
        LOCATE 23, 10
        PRINT "Any key to continue."
        a$ = readkey$
    END IF
NEXT p%
END SUB

'Sends the summary data to the print file 'ndiaph.sum'.
    IF (count > 1) THEN PRINT #2, CHR$(12)
    p$ = LTRIM$(STR$(profile))
    PRINT #2, TAB(30); "SUMMARY OF "; p$; " PROFILES."
    CALL sumconsts(mtitle$, name$) 	 'file constants
    CALL lines(2, 1)
    PRINT #2, TAB(30); "Diaphragm diameter = ##.## mm";
    PRINT #2, TAB(10); "True stress at pole = #.####——MPa"; pstress
    PRINT #2, TAB(10); "SED at pole = #.####——MJ/m^3"; energy
    END IF
    IF (pfile = 1) THEN CALL filedat(p%, lamc, mtitle$, pstress, energy)
    END IF
    IF (disp = 1) THEN
        LOCATE 23, 10
        PRINT "Any key to continue."
        a$ = readkey$
    END IF
NEXT p%
END SUB
SUB
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PRINT #2, USING "thickness = $#### mm"; thk
PRINT #2, TAB(10);
PRINT #2, USING "No. of increments = #"; nn

CALL lines(2, 2)

PRINT #2, TAB(10); "Pres. Height";
PRINT #2, TAB(30); "Width at Ht.";
PRINT #2, TAB(50); "Lambda Stress S.E.D."
PRINT #2, TAB(10); "(kPa) (mm)"
PRINT #2, TAB(30); "(mm)"
PRINT #2, TAB(50); "(MPa) (MJ/m^3)"

PRINT #2, ""
FOR k% = 1 TO profile
PRINT #2, TAB(10);
PRINT #2, USING "###.##"; apres(k%); ht(k%); wd(k%); wdh(k%);
PRINT #2, TAB(50); "Lambda"; lamax(k%); pstr(k%); psed(k%)
NEXT k%

CALL lines(2, 2)
PRINT #2, TAB(10); "Pres. Crown";
PRINT #2, TAB(30); "Half Half";
PRINT #2, TAB(50); "Volume Energy"
PRINT #2, TAB(10); "(kPa) Curv.";
PRINT #2, TAB(30); "Height Width";
PRINT #2, TAB(50); "(mm) (J)"
PRINT #2, TAB(10); ";(mm)"
PRINT #2, TAB(30); "(mm)"

PRINT #2, ""
FOR k% = 1 TO profile
PRINT #2, TAB(10);
PRINT #2, USING "###.##"; apres(k%); crcrv(k%); hlfht(k%); hlfwd(k%);
PRINT #2, TAB(50); "Volume"; vol(k%); twork(k%);
NEXT k%

END SUB

FUNCTION readkey$
'Accepts a single key.
'See RUPEC.BAS
END FUNCTION

SUB scrtitle
'The title screen for the program.

SCREEN 7
COLOR 15, 1
CALL headframe(2, 2, 38, 23)
LOCATE 4, 4
PRINT "RuPEC";
LOCATE 5, 4
PRINT "Loughborough";
LOCATE 6, 4
PRINT "University";
LOCATE 4, 28
PRINT " RUBBER";
LOCATE 5, 28
PRINT "ELASTICITY";
TL$ = "SIMULATION OF"
T2$ = " INFLATED"
T3$ = " DIAPHRAGM"
LOCATE 12, 8
PRINT TL$;
LOCATE 14, 8
PRINT T2$;
LOCATE 16, 8
PRINT T3$;
LOCATE 18, 8
PRINT "JFH/PSO";
CALL delay(2)
END SUB
SUB setmenu (item$())
'Contains the main menu items.
FOR i% = 1 TO 11
   item$(i%) = ""
NEXT i%
item$(1) = "Results Directory and Files"
item$(2) = "Diaphragm Specification"
item$(3) = "Original Turner Constants"
item$(4) = "Modified Turner Constants"
item$(5) = "Modify F only"
item$(6) = "Maximum Height"
item$(7) = "Specific Extension Ratio"
item$(8) = "Output"
item$(9) = "Calculate Profile(s)"
item$(10) = "Display Profile(s)"
item$(11) = "Quit"
END SUB

SUB sumconsts (mtitle$, name$)
'Writes the modified Turner elastic constants to, first, the summary file
'and then the Excel file.
CALL lines(2, 2) 'titles
PRINT #2, TAB(10); "Title: "; mtitle$ 'summary
PRINT #2, TAB(10); "Data file (.tnr) reference: "; name$
CALL lines(2, 1)
PRINT #2, TAB(10); "Elastic Constants (MPa):"
fmt$ = "#11.###^^^^
CALL lines(2, 1)
PRINT #2, TAB(15); "Initial tension: ";
PRINT #2, USING fmt$; tcnst(1);
PRINT #2, TAB(15); "Linear coeff.: ";
PRINT #2, USING fmt$; tcnst(2);
PRINT #2, TAB(15); "Quadratic coeff.: ";
PRINT #2, USING fmt$; tcnst(3);
PRINT #2, TAB(15); "Cubic coeff.: ";
PRINT #2, USING fmt$; tcnst(4);
PRINT #2, TAB(15); "Quartic coeff.: ";
PRINT #2, USING fmt$; tcnst(5);
PRINT #2, TAB(15); "Quintic coeff.: ";
PRINT #2, USING fmt$; tcnst(6);
PRINT #2, TAB(15); "Eguibiax.factor: ";
PRINT #2, USING fmt$; tcnst(7)
'Now the Excel file.
extitle$ = "Title: " + LTRIM$(mtitle$) + "{" + LTRIM$(name$) + "}"
PRINT #4, extitle$
PRINT #4, "Elastic Constants:"
fmts = "#.#####";
PRINT #4, "Init.tension,Linear coeff.,Quad.coeff.,Cubic coeff.,Quart.coeff.,Quin.coeff.,Eguibiax.factor."
FOR i% = 1 TO 6
   PRINT #4, USING fmt$; tcnst(i%);
NEXT i%
PRINT #4, USING "#.####"; tcnst(7)
PRINT #4, ""
END SUB

SUB summary (mtitle$, name$)
'Displays a summary of the calculated profiles.
CALL surround(2)
COLOR 0
p$ = LTRIMS(STR$(profile))
LOCATE 3, 30
PRINT "SUMMARY OF "; p$; " PROFILES."
LOCATE 5, 8
PRINT "Pres. Height Width at Ht. ";
PRINT " Lambda Stress S.E.D"
LOCATE 6, 8
PRINT "(kPa) (mm) (mm) (mm) "

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PRINT " (MPa)  (kJ/m^3)"

1% = 7
FOR k% = 1 TO profile
  1% = 1% + 1
  LOCATE 1%, 8
  PRINT USING "###.#  ###.#"; apres(k%); ht(k%);
  LOCATE 1%, 28
  PRINT USING "###.#  ###.#"; wd(k%); wdh(k%);
  LOCATE 1%, 51
  PRINT USING "###.#  ###.#"; lamax(k%); pstr(k%);
  LOCATE 1%, 69
  PRINT USING "###.#"; psed(k%);
NEXT k%

'Calculate and plot chosen height details

1% = 1% + 1
pl% = profile - 1: p2% = profile
profile = profile + 1
delta = (maxht - ht(pl%)) / (ht(p2%) - ht(pl%))
apres(profile) = apres(pl%) + (delta * (apres(p2%) - apres(pl%)))
ht(profile) = ht(pl%) + (delta * (ht(p2%) - ht(pl%)))
wd(profile) = wd(pl%) + (delta * (wd(p2%) - wd(pl%)))
wdh(profile) = wdh(pl%) + (delta * (wdh(p2%) - wdh(pl%)))
lamax(profile) = lamax(pl%) + (delta * (lamax(p2%) - lamax(pl%)))
pstr(profile) = pstr(pl%) + (delta * (pstr(p2%) - pstr(pl%)))
psed(profile) = psed(pl%) + (delta * (psed(p2%) - psed(pl%)))

k% = profile
LOCATE 1%, 8
PRINT USING "###.#  ###.#"; apres(k%); ht(k%);
LOCATE 1%, 28
PRINT USING "###.#  ###.#"; wd(k%); wdh(k%);
LOCATE 1%, 51
PRINT USING "###.#  ###.#"; lamax(k%); pstr(k%);
LOCATE 1%, 69
PRINT USING "###.#"; psed(k%);

CALL work
CALL prntsum(\"name\")
CALL excel
COLOR 4
LOCATE 25, 31
PRINT \" Any key to continue \"
a$ = readkeys
CALL surround(1)
END SUB
Appendix F

Temperature Change Example Calculation.

F.1 Introduction.

Although not deemed to be a problem during this work, the compression and/or expansion of the air used in the biaxial tests may cause undesirable temperature changes in the polymer. In this Appendix a very simple example of adiabatic pressure change is included to indicate the possible air temperature change. However, this is only the theoretical static change in temperature, and mass flow effects should also be considered to give a more accurate indication. Due to the relatively low heat conductivity from air to rubber, the change in rubber temperature will also be less severe than calculated. Both of these factors however are outside the scope of this thesis.

F.2 Example Calculation.

For an ideal gas the following hold:

\[ PV = nRT \]  

\[ PV^\gamma = \text{constant} \]  

where \( P \) is the pressure, \( V \) the volume, and \( T \) the temperature. \( R, n \) and \( \gamma \) are constants, of which \( \gamma = 1.4 \).

Now at the start of the test the air line pressure, approximately 3 bar in the fatigue test, will have dropped to just above ambient, assumed here to be 1.1 bar. From equation F.2;

\[ \frac{P_2}{P_1} = \left( \frac{V_1}{V_2} \right)^\gamma \]  

and inserting the appropriate values;

\[ \frac{11}{3} = \left( \frac{V_1}{V_2} \right)^\gamma \]
hence;

\[ \frac{V_1}{V_2} = 0.367^{\gamma} \]  \hspace{1cm} \text{F.4}

Now from equation F.1;

\[ \frac{P_1 V_1}{P_2 V_2} = \frac{nRT_1}{nRT_2} \]  \hspace{1cm} \text{F.5}

Simplifying, and inserting the result from equation F.4 yields:

\[ \frac{3}{1.1} \cdot 0.367^{\gamma} = \frac{T_1}{T_2} \]  \hspace{1cm} \text{F.6}

Now assuming the start temperature is 293K (20°C) as the equipment is well down stream of the compressor, the final temperature from equation F.6 is 219.8K (-53°C).